THE PHILOSOPHICAL VIEWS OF ERNST MACH.[1]

AS a preliminary characterization of Mach's thought, let us answer in his spirit the question: What is the task of philosophy? It is the function of philosophy to coördinate and organize the special sciences into a whole.[2] To this end it is the function of philosophy to interpret one special science in the light of another and by this comparative interpretation to gain critical points of view. Philosophy should aim to be the unification and interpenetration of the special sciences and nothing more.[3] Philosophy is approximately what Comte understood by the systematization of the particular sciences. Philosophy cannot transcend the special sciences and make discoveries in unknown lands outside their content; it should hold fast to the 'positive,' the 'given,' as the separate disciplines do or ought to do. The standpoint of the 'given' is deserted and all effort is fruitless, as soon as the thinker attempts to do more than to systematize and correlate the deliverances of the particular sciences, to bring these deliverances into mutual relationship, and in this way mutually to supplement and clarify them. Mach's conception of philosophy, like Comte's, is positivistic.

What do we understand by the 'given'? Mach answers that the 'given' consists in appearances, phenomena. One must not, however, include in the notion of phenomena any reference to something behind the phenomena, as Kant does. Appearances or phenomena are, for example, the sensations: colors, tones, heat, pressure, also space and time. We must also include feelings among phenomena. Philosophy cannot go beyond these 'given elements,' and should have no disposition to go beyond

[1] In citing Mach's works the following abbreviations are used: *E. d. A.* = *Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit*, Prag, 1872; *B. z. A. d. E.* = *Beiträge zur Analyse der Empfindungen*, Jena, 1886; *A. d. E.* = *Die Analyse der Empfindungen und das Verhältniss des Psychischen zum Physischen*, Jena, 1900; *W.* = *Die Principien der Wärmelehre*, Leipzig, 1900; *M.* = *Die Mechanik in ihrer Entwickelung*, Leipzig, 1901; *P. V.* = *Populär-wissenschaftliche Vorlesungen*, Leipzig, 1903.

[2] *A. d. E.*, p. 208.

[3] *P. V.*, p. 289.

them. In this respect Mach's conception of philosphy is phenomenalistic. That, at least, is the aim of his philosophy, and, for the most part, though not always, that is its character. At times, however, there appears behind the movement of phenomena a metaphysical, Schopenhauerian Will. In general, Mach's thought is anti-metaphysical, positivistic.

In the 'given' we have colors, tones, temperature, space, and time, interrelated in manifold ways; and with them are joined moods, feelings, and volitions.[1] "The relatively more fixed and constant arrests attention in this complex; it impresses itself on the memory and finds expression in language. As the relatively constant, we notice first such complexes of color, tone, pressure, etc., as are spatially and temporally conjoined, and to which, consequently, particular names are given; they are called bodies.[2] Bodies are merely relatively constant sensation-complexes. A table is a totality of spatially arranged contents of visual sensation; also, it may be, of tactual sensation. This totality contains a large constant element; the top is always black, the feet are brown; in the case of touch, I have constantly the sensation of something hard, of firm resistance, etc. But the table is by no means an absolutely constant sensation-complex. The brown color of the feet is sometimes brighter, sometimes darker, depending on the amount of light by which they are illuminated; the top feels sometimes warmer and at other times cooler.

As bodies are seen to be only relatively constant in the shifting of sensation, so also the ego is only relatively constant. The ego is a complex of memories, words, feelings, that are conjoined with a definite body (my body). I can do this or that; I can be quiet, joyful, sad, or aroused. Always, however, there remains "enough of the constant for me to recognize it as the identical, persistent ego."[3] The apparent constancy of the ego consists "chiefly in its continuity," in the steady, slow change. The remnant of moods, feelings, and memories that persist from childhood to old age is little enough.

Absolutely constant complexes are not found amongst sensations, memories, feelings, nor in the will. The notion of sub-

[1] *A. d. E.*, p. 1. [2] *Ibid.*, p. 2. [3] *Ibid.*, pp. 2 f.

stance, the unchangeably persistent, arises from our experience of the constancy of complexes. Changes in the complexes give rise to the notion of attributes, which have the power of changing without affecting the immutability of the substance. A fruit is sweet, but it may be bitter. Also other fruits are sweet. Many bodies have the same color. A piece of iron may be hot or cold. By burning it may cause me pain, or it may when cold give me a pleasant sensation of coolness. By such experiences we learn to separate sensations from substances,[1] and we thus form the conception of a persistent substance with variable properties. The constant is designated by a single name and by custom is held together in thought. It acquires an existence for itself apart from attributes, whose mutability is apparent. We can think each attribute of a thing, one by one, as abstracted from the thing itself. And so we think we may finally remove all the attributes of a thing without removing the thing. There arises thus the notion of a thing-in-itself, behind the attributes, which are only phenomenal. We can know only the phenomenal attributes; the thing *per se* remains unknown behind the appearance.

The problem of a thing with many attributes is a product of this connexity in thought. If one had only noticed how the thing-in-itself comes into being, and had, with that process in mind, held fast to the fact that in complexes everything is merely relatively constant, that there is no absolutely constant nucleus in sensation-complexes, this whole supposed problem would have fallen to the ground along with the monstrous notion of a thing-in-itself. It is not true that there is a thing possessed of a higher reality endowed with properties possessed of a lower reality; as a matter of fact, there are only real sensations, which form relatively constant complexes, bodies. The real elements are the sensations.

Similar to the conception of a single thing with manifold attributes, we have the conception of a single soul with manifold expressions. Here, too, the spurious problems with which Herbart labored, have arisen in a similar way. Let A, B, C be

[1] *Op. cit.*, p. 4.

the sensations, whose complex we are accustomed to call body. Let *K*, *L*, *M* be the sensation-complex which we call the human body. The latter complex is distinguished from the former in many ways. Let us call the complex of our common feelings, moods, volitions, and memories, α, β, γ. The naïve observer opposes the complex α, β, γ . . . *K*, *L*, *M*, the ego, to the complex *A*, *B*, *C*, the non-ego, *i. e.*, other things. Sometimes, too, α, β, γ, as the ego, is opposed to *K*, *L*, *M* . . . *A*, *B*, *C*, as the corporeal world. "In the first place, *A*, *B*, *C* appears to be independent of the ego and to stand opposed to it as a separate entity. This independence is only relative and will not stand close examination. In the complex α, β, γ, considerable change may, indeed, take place without such change being noticeable in *A*, *B*, *C*, and conversely. Many changes, however, in α, β, γ pass into *A*, *B*, *C* through changes in *K*, *L*, *M*, and conversely (when, *e. g.*, vigorous ideas pass into actions or the environment causes noticeable changes in our bodies). In this case, *K*, *L*, *M* appear to have more intimate connection with α, β, γ and also with *A*, *B*, *C* than the latter have with each other."[1] Hence the isolated position of the complex *K*, *L*, *M* (my body), amongst the complexes *A*, *B*, *C*, *i. e.*, amongst bodies in general. Hence also the opposition of the complexes α, β, γ . . . *K*, *L*, *M*, as the ego, to the complexes *A*, *B*, *C*, or the designation of α, β, γ as the ego in contradistinction to the complexes *K*, *L*, *M* . . . *A*, *B*, *C* as the corporeal world.

On closer examination it is seen "that *A*, *B*, *C* is always co-determined with *K*, *L*, *M*. A die, when seen near to the eye, looks large, when at a distance, small; with the right eye otherwise than with the left, at times double, and with closed eyes it is not seen at all."[2] The complexes *A*, *B*, *C*, then, depend very much on the complexes *K*, *L*, *M*. The attributes of bodies (*cf.* the die referred to above) appear modified, or conditioned by our body. A more adequate statement of what actually takes place would be that "the various *A*, *B*, *C* complexes are bound up with the various *K*, *L*, *M* complexes."

Let us take the example of a needle. In visual sensation we

[1] *Op. cit.*, pp. 6 f. [2] *Ibid.*, p. 7.

have the point *P*. If *P* touches the end of my finger, a tactual sensation and feelings are added to my visual sensations. If *P* is removed from my finger, only visual sensations remain. In this case, the visual sensation appears as the persistent (the constant), compared with the tactual sensation (the accidental). But the visual sensation is also not constant; it is accidental, like the tactual sensation. If we remove the needle from the field of vision, all perceptions of the needle vanish. After such disappearance nothing remains in experience, no thing-in-itself.

The elements *A*, *B*, *C* stand to α, β, γ quite as much in reciprocal dependence as do *K*, *L*, *M*; only the relations between α, β, γ and *K*, *L*, *M* are more numerous and relatively more constant. The difference is one of relativity. When I contemplate the dependence of *A*, *B*, *C* on α, β, γ, I conclude that *A*, *B*, *C* belong to me, are my sensations. "The ego can, therefore, be expanded to include ultimately the entire world."[1] In any case, however, we must keep constantly in mind the relativity of the distinction in complexes. As helps for immediate orientation, the notions ego, my body, and other bodies, are useful.

It is not the business of science to explain the existence of the elements α, β, γ . . . *K*, *L*, *M* . . . *A*, *B*, *C*, but to recognize these as the ultimately 'given' in experience.

The notion of substances loses all philosophical meaning for Mach, because in the complex of 'given' elements there is nothing absolutely persistent, but only something relatively constant. For him the world is a totality of elements, *i. e.*, of sensations, memories, feelings, and volitions. These elements are interrelated in manifold ways. It is the function of science to determine these connections and interrelationships.

How is this to be done? In the first place, it is evident that we must not refer relations between the given elements, *e. g.*, sensations, to relations between substances. For it is not permissible, as we have seen, to admit the existence of such substances. If, however, we conceive the elements and their relations as dependent on substances, we conceive them as caused by substances, by things-in-themselves. In this application the

[1] *Op. cit.*, p. 8.

notion of effect, and correspondingly the notion of cause, is, of course, false.

But the notion of cause in none of its forms withstands positivistic criticism. It is a useful notion for the common needs of life, as is also the notion of substance. But it does not meet the demands of scientific thought. The taint of vulgar origin clings to the twin notions of cause and effect. However abstractly we may interpret these notions, they cannot conceal the fact that they had their origin in animistic ideas. Let us examine this origin.

"In general, we feel the need of inquiring after a cause only where some unusual change takes place. The familiar changes in our environment are those which are produced by our own will, which fact leads to the conception of animism and fetichism." [1] The naïve man confidently transfers the peculiar interrelation which he has experienced a thousand times in his own acts of will, to the changes in external nature. Processes in nature that attract his attention appear to him in the light of acts of will, exerted by spiritual beings concealed in or behind natural things. Looked at logically, animism involves a very doubtful conclusion from analogy. And it finds an intellectual satisfaction in this analogical process; it feels that it has gained insight into the processes of nature. For in its consciousness the connection between an act of will and movement is so familiar that it appears self-evident. In the animistic explanation of nature's processes, something of this self-evidence is transferred to antecedent and consequent in nature. If this connexity is self-evident, it also appears to the human mind as necessary. The self-evident sequence becomes a necessary sequence,— a sequence necessary not merely for thought. What is necessary for thought appears at once as also necessary for the real.

Popular belief, as is well known, sees preferably a connexity in such things as exhibit a certain, although frequently very external, similarity. "In this way the fruits of plants are regarded as curative for the head; roots, on the contrary, as curative for the feet, etc. For extraordinary effects one seeks quixotic causes, of which the witches' broth in Shakespeare's *Macbeth* furnishes a

[1] *W.*, p. 432.

striking example." [1] The cause, in this popular conception, must have kinship and similarity with the effect; a partial sameness must be discoverable in cause and result. Things that have nothing in common with each other cannot affect each other.

How far has this popular conception of causality penetrated into philosophical, and, in general, into scientific thought! One does not need to revert to the old philosophers of nature; one has only to recall Descartes and Spinoza. Things which have nothing to do with one another, like extension and thought, have no reciprocal action! But an extended thing may produce an effect on an extended thing, and a psychical thing may affect another psychical thing.

All such conceptions of causality must vanish in the face of positivistic criticism. "Hume's analysis, his commentary on the case of the man lame in his arm, who, in spite of his will, is unable to move his arm, is an excellent instance for a higher critical point of view." [2] "In reference to the connection between will and movement, we have no more insight than we have in reference to any other connection, in the opinion of Hume; he concedes ultimately only the validity of expectation based on custom." [3] Hume's criticism hits the truth. The given is always merely the actual sequence. This may become familiar, self-evident. Then the subjective compulsion of thinking cause and effect together, mirrors for us an objective necessity in the external complex. Other combinations are not so familiar; they appear, therefore, unnecessary.

Hume's criticism is also correct in those instances where cause and effect have some similar or kindred element, or where a causal dependence between the same or similar things is observed. That, in the case of impact, the motion of the striking ball is transferred to the ball which is struck, is known only by the experience of the actual history of the process. Here, however, the connexity between the phenomena is particularly simple and observable. The difference between this and more complicated causal connections is a "difference merely in degree, which falsely presents itself as a qualitative difference [in comparison with a

[1] *Op. cit.*, p. 433. [2] *Ibid.*, p. 432. [3] *Ibid.*

less readily observed instance]."[1] "In many instances we scarcely think of the possibility of a connection, while in other cases we are in a position of mental compulsion, and in the latter cases the connection seems necessary. To the artillerist, *e. g.*, the flight of a projectile appears necessarily dependent on its initial velocity and direction. As a matter of fact, if the event corresponds to known and simple geometrical (phoronomic) laws, then the event is quite as clear to us as those laws; initial velocity and initial direction then become the basis of knowledge, from which the flight of the projectile is logically and necessarily derivable. In the moment, however, of feeling this logical necessity, we do not at once think that the existence of those conditions is given simply in our experience, without being in the least based on necessity."[2] Any physical necessity, or any necessity other than a logical one, is denied by Mach.

"The relation between ground (of knowledge) and consequence is also demonstrably Kant's ideal"[3] in his theory of causality. Here Mach holds with Hume against Kant. One is concerned with a concept developed within experience itself.

It is true that one may accept Hume's view of causality and still retain the notions of cause and effect as scientifically serviceable. John Stuart Mill, starting from Hume's standpoint, defended them emphatically against the criticism of Comte. Mach's doubts concerning them are deeper. Such doubts as to the validity of the notion of causality find expression in the following example. Let us take two masses, m_1 and m_2. "When we speak of the 'attraction of masses,' it might appear that this expression contained more than the actual. But whatever we add over and above the actual is certainly idle and useless."[4] If we ascribe to masses powers of attraction, which we regard as really existent, we add an idle and useless element. Further, if we regard one mass as the cause of the motion of another mass, we do not give a satisfactory account of the actual facts. If, however, we describe the reciprocal acceleration as $\varphi = \frac{m_1 + m_2}{v^2}$, we give by the formula an account of the entire history of the phenom-

[1] *Op. cit.*, p. 434. [2] *Ibid.* [3] *Op. cit.*, p. 435. [4] *Ibid.*

enon, and we avoid all superfluous additions. A report of the relations and processes furnishes a much more adequate account than the mere citation of causes.

The citation of cause not only fails to furnish what ought to be furnished; it is itself something in the highest degree manifold and undetermined. What is the cause of the acceleration of the two bodies? Is it the one mass? No, the motion depends quite as much on the other mass. Is it both masses? Or is it both masses and their spatial positions? Their spatial positions at what moment? Or, to take another favorite example: An avalanche rolls into a valley and destroys a building. What is the cause of the destruction? The avalanche? Or is it the little particle that perhaps started the avalanche? Is it the kinetic energy of the avalanche? Is it the unfavorable situation of the house, or the frailty of its structure? The citation of a single cause for an actual process is in the highest degree arbitrary, ambiguous, and scientifically of little use.

"If one attempts to do away with all traces of fetichism that cling to the notion of cause, and considers the fact that a *single* cause cannot, in the rule, be given, that, on the contrary, any given event is generally determined by a whole system of conditions, one is led to give up entirely the concept of cause. It is better to regard the notional determining elements of a fact as dependent on each other, in just the same way as the mathematician or, let us say, the geometrician does."[1] As the sides and angles of a triangle stand in reciprocal dependence on each other, so the elements, *e. g.*, sensations, etc., stand in vastly more complicated relations of dependence.

Geometrical relations admit of calculable comprehension, of being presented in a garb of analytical statement. The dependence of distances, coördinates, etc., is analytically representable as a functional dependence of mathematical magnitudes and variables. Similarly, we can represent the dependent relations of elements, of sensations, let us say, by means of functional references. "For this reason I made the attempt long ago to substitute the notion of function for the notion of cause."[2] "The

[1] *Op. cit.*, p. 436.

[2] *A. d. E.*, p. 66. *Cf. E. d. A.*, 1872.

ghost [of metaphysical difficulties] immediately disappears, if one conceives the situation in a mathematical sense, and becomes clearly aware that only the mediation of functional relations has any value for us, that all we want to know is merely the interdependence of experiences. We then see that reference to unknown, not given, primary variables (things-in-themselves) is entirely fictitious and idle." [1]

Let us return to the example of the attraction between two masses. The citation of causes and effects gave very little information. The explanation of the acceleration, however, as a function of the masses, answers every question the physicist can raise relative to the course of facts. Physics is cognizant of a great many such functional relations. If the problem has to do with a temporally determined course of facts, time will appear in the function as a variable. If the problem is concerned with synchronous relations, time will appear in the function as a constant. When a physicist investigates a hitherto unknown process of nature, inquires into it qualitatively, he merely looks for functions appearing between measurable magnitudes, which functions manifest themselves in the process. "Our investigation aims at equations that exist between the elements of phenomena. This method is transferable to every form of scientific procedure as its ideal. Relations of dependence are not in all cases sufficiently simple and determinable to be capable of mathematical statement. Nevertheless, the search for such dependences remains the aim of all scientific inquiry. The distance of the various disciplines from this goal is very different. Lagrange brought mechanics very near the ideal by universalizing the method of Euler and Maclaurin, the method of analysis, and by applying it to the whole of mechanics." [2]

In functions which set forth the interdependence of the elements, the coördinates of place and time play a rôle that corresponds entirely with the other variables. They are variables along with the others; for they are also referable to elements that stand in manifold relations to each other, viz., to perceptions of time and space. The law of causality emphasizes the

[1] *A. d. E.*, p. 25. [2] *M.*, p. 496.

temporal element in equations as something *sui generis*, to which Mach cannot assent. "Peculiar emphasis on space and time in the law of causality is unnecessary; for all temporal and spatial relations are themselves referable to interdependences of phenomena." [1] "We can eliminate time from every natural law by substituting for it the earth's angle of revolution." [2] Functional dependences, "these equations or relations, are, properly speaking, the constant." [3] Constancy of dependences of relations determined by law, takes the place (in Mach's thought) of constant substances, of immutable things-in-themselves. The great Greek thinkers sought in the flux of phenomena preëminently constant things, immutable being, the substantial; the development of modern philosophy, especially progressive natural science, has laid great emphasis on the immutability of law, while retaining the conception of abiding substances, things-in-themselves, atoms. To complete the development, one must go farther on this course. Mach's positivism rejects entirely the constancy of substantial elements, inner nuclei of phenomena, and lays greater stress on the constancy of laws.

"One may claim as evidence of the existence of an extra-perceptual substantial condition of sensation, that a body which I perceive in a certain way must also be perceived by others in a corresponding way." [4] As a matter of fact, one explains the agreement in the various perceptions of a body by the theory of a substance independent of sensation, a thing-in-itself which affects several egos and produces in them like sensations. For Mach's positivism this circumstance signifies "merely that equations similar to those which exist between the more closely coherent elements, and which constitute my ego, also exist between the elements of other egos, I′, I″, I‴, . . . whose ideas facilitate my understanding of the world, and further that there are comprehensive equations which include the elements of the other egos I′, I″, I‴. . . . In this connection, more than the above will not be accepted by an investigator who wishes to avoid spurious problems." [5]

[1] *Op. cit.*, p. 536. [2] *E. d. A.*, p. 35. [3] *W.*, p. 424.
[4] *Ibid.* [5] *Ibid.*

Mach has, nevertheless, introduced a metaphysical conception into his view of causality and of that which exists in itself. Beneath the movement of sensations and feelings that express themselves in individuals, we see a directing Will, seeking pleasure, avoiding pain. "Preservation of the species is, on the whole, merely a factual and valuable postulate for investigation, but by no means the final or highest. In reality, species have perished and new species have undoubtedly arisen. The pleasure-seeking, pain-avoiding Will must have a more far-reaching significance than the preservation of the species. It preserves the species when it is worth while; it destroys it when the continued existence of the species is no longer advantageous. If the will were directed merely to the preservation of the species, constantly deceiving all individuals and itself, it would move aimlessly in a fallacious circle."[1] The Will which cares for the species, preserving or destroying it on grounds of what is advantageous or disadvantageous for the pleasure-seeking Will, cannot be the individual, empirical will. The individual will aims almost without exception at the preservation of the individual, and so of the species. The Will described by Mach must be something that transcends the individual will and dominates the will-movement of the entire species. It disposes of species in accordance with its own ends. The intellect is something secondary, ancillary, to the will. "Ideas are not the whole of life. They are merely fugitive, illuminating visions, intended to light the path of the will."[2] One finds in Mach confirmation of Schopenhauer's remark that the Will created the intellect for its own ends.

This is voluntaristic metaphysics. The empirical will never created the intellect for its ends. The empirical will presupposes intellectual functions in Mach's philosophy. It is "nothing but the totality of partially conscious conditions of a movement, conditions coupled with prevision of result." In the second and enlarged edition of the *Beiträge zur Analyse der Empfindungen* (1900), in the analysis of sensations, etc., Mach added to the above statement the following: "One can very well accept Schopenhauer's conception of the relation between will and

[1] *B. z. A. d. E.*, pp. 38 f. [2] *P. V.*, p. 219.

force, without seeing in them anything metaphysical."[1] One must, however, characterize the statement in question as metaphysical, unless one arbitrarily changes the definition of metaphysics. In reference to the parallelism between organic and inorganic processes, Mach says that in inorganic nature one finds something "somewhat analogous to will," following, however, much simpler laws. Our hunger is "not so very essentially unlike the attraction of zinc for sulphuric acid, and our will is not so different from the pressure of a stone on its substructure"[2] as one is at present disposed to believe. Naturally the ascription of complete personality to a tree or stone is for us an unjustifiable deduction. Animism, however, was not totally wrong in fancifully ascribing to trees and stones a spiritual being; it simply went too far. It sought for personalities behind the tree or stone, whereas it should have argued only to "something analogous to will."

Unquestionably Mach assumes in these passages a genuine metaphysical reality, which corresponds to the bodies of my sense perception, yet the contradiction between this and his antimetaphysical explanation is perhaps not so great as might appear. Unfortunately, Mach has done little to facilitate an adjustment between the two sets of ideas.

Evidently we must interpret the situation as follows. From the elements, the 'given,' viz., from my sensations, feelings, and volitions, I arrive by analogy at the sensations, feelings, volitions of other individuals.[3] The conclusion from analogy receives, however, a wider application; it is applied to the higher and lower animals, plants (the tree cited by Mach), and finally to the stone, in inorganic nature. The reasoning from analogy becomes all the while less precise, the conclusion more indefinite. But, in the philosophy of Mach, who accepts the doctrine of evolution, analogy persists all the while. Everything that we conclude from analogy must be somehow regarded as like that from which our analogical reasoning starts. The starting point is the 'given,' the elements, which I must characterize as mine.

[1] *A. d. E.*, p. 61. *Cf.* the cautious remark on p. 226.

[2] *M.*, p. 439.

[3] *Cf. W.*, p. 424, cited above.

Every conclusion, therefore, must represent a totality of elements, or complexes of sensations, feelings, volitions, or complexes of similar elements of greater or less complication. In this way I arrive at a self-existent something in contradistinction to my elements. Yet this self-existent something is quite as little persistent in its complexes as are the complexes of my elements. In these complexes based on analogy we have also only the persistence of laws. Persistence of laws obtains also between elements of diverse complexes; in a word, between all existing elements. In this way, Mach could logically maintain his criticism of the notion of substance, the notion of existential nuclei in contradistinction to that of the elements, and also his criticism of the notion of causality, in spite of his metaphysical postulates. These are elements which are combined in far simpler complexes than those recognized as the totality of our own personal and spiritual life.

This metaphysic is quite reconcilable with the doctrine of evolution, which Mach adopts. The complicated totalities of elements which form my ego and the egos of other men, are derived from simpler totalities of elements, the lower animals. If the series is followed back, one comes to complexes of elements more and more simple. These in turn must have in reference to my sensations an independent existence, just as the sensations of an animal have, in reference to my sensations of this animal, an existence *per se*. Again, one arrives at the above result thus: Correlated with my perceptions of bodies there are, *per se* existent, more or less simple complexes of sensations or will.

The theory of evolution is important for Mach's philosophical views in another direction. It influences deeply his conception of the nature and task of scientific investigation. The distinction between human and animal intelligence is not one of quality, but of degree. "I am of the opinion that the view which makes a qualitative distinction between human and animal intelligence is a remnant of an old superstition. I can see merely a quantitative distinction, a difference of degree, in the order of animals (including men), a difference which widens with increasing sepa-

ration in organization. . . . A similar distinction is observable between the child and the adult. Further, I see only a quantitative distinction between human and animal language." [1] The development of intelligence is a partial phenomenon in the general process of evolution. In Mach's opinion, "evolution of thought is a part of the general development of life, the adaptation to an increasing range of activity." "Knowledge is an expression of organic nature." This holds in the first place for practical knowledge. It holds, however, not merely for this; all knowledge "issues ultimately from the demands of practical living," [2] from foresight for the future, from technic. Geometry developed from surveying, astronomy from agricultural and nautical needs, chemistry from metallurgy and alchemy. "In general, the mental activity of the investigator is not so different from that of ordinary life as one commonly supposes." [3] The aim of science and the aim of the knowledge of everyday life are, therefore, primarily not different. Science is merely a completion of practical knowledge. Looked at 'biologically,' science has the task of offering to man 'the most complete orientation.' We need a world-view, a cosmic map, to find our bearings in our environment.

Yet Mach concedes a more far-reaching significance to theoretical knowledge. "The intellect, strengthened by work in foreign service, soon makes its own needs felt." In this way there arises a double task for investigation. On the one hand, it furnishes the basis of technic. On the other hand, it exists for the sake of knowledge itself, or for "the satisfaction of intellectual unrest." [4] Science has partly freed itself of its first task, — the task of satisfying practical needs. Meanwhile, even in the higher stages of culture, these elementary processes of knowledge, which serve natural life, form a strong foundation for scientific thought. "Half consciously and involuntarily," "instinctively," this knowledge has been won by man. "This instinctive knowledge, because of the conviction that we have consciously and voluntarily contributed nothing to it, confronts us with an authority and logical power that knowledge which is consciously and vol-

[1] *W.*, p. 410. [2] *M.*, p. 541. [3] *P. V.*, p. 16. [4] *A. d. E.*, p. 209.

untarily acquired, even from the best known and most easily tested sources, cannot attain. All so-called axioms are of this instinctive character." [1]

Primary experiences on this lower plane of intellect continue to this day to be important. Many such elements of knowledge constitute material presuppositions for the particular sciences. Old knowledges become axiomatic; others may become the 'forms' of thought. Take the law of causality as a case in point. What the individual man gains through experience and the training of insight, may perhaps be inadequate to account for the compulsion he feels to seek for causal relations in the flux of phenomena. In the spirit of Spencer, Haeckel, and Hering, Mach is of the opinion that "many of the forms of thought are not arrived at by the individual, but are preformed, or at least prepared, through the development of the species." [2]

As we find in the instinctive elements of knowledge, in axioms and the forms of thought, a survival of the childhood's stage of knowledge, so the origin of science from practical insight into materially advantageous ends is revealed in the economic nature of investigation. "Is not science itself a business? Is not its aim the attainment of the greatest amount of eternal, infinite truth, with the minimum expenditure of labor, in the minimum of time, and even with the minimum of thought?" [3] The entire apparatus of scientific investigation is comparable with the system of production in an industrial enterprise. Here, as there, we observe the same phenomena: the increasing division of labor, which provisionally is merely regulated and organized by means of corporations, etc. On the whole, however, the principle of *laissez faire* dominates both scientific and material production. It is the function of science to think the world with the minimum measure of energy (Avenarius).

New light is thrown on the anti-metaphysical tendency of Mach's thought by his view of the economic nature of scientific investigation. "If one starts with the economic task of science,

[1] *P. V.*, p. 219.
[2] *Ibid.*, p. 251; *cf.* also *P. V.*, p. 219, and *M.*, p. 514.
[3] *Ibid.*, p. 16.

as I have done, according to which only interconnections within the observable, the 'given,' are significant, whereas everything hypothetical and idle is to be eliminated, one arrives at this [anti-metaphysical] position. The same position would probably be ascribed to Avenarius."[1] Especially the idea of a substance, a nucleus, is to be rejected. This idea contributes nothing to the completeness of our orientation; for the interconnections between sensations are not simplified by importing into the function which represents these interconnections, the fundamental variable of the thing-in-itself. For this reason the concept of function is preferable to the concept of cause. If, in the case of a falling body, one states the cause, viz., the power of attraction between masses, etc., one cannot in this way furnish nearly so complete and brief an orientation concerning the phenomenon of gravity as by giving the function, the formula for falling bodies: $s = \frac{1}{2}gt^2$. The explanation by means of the notion of cause and effect is too cumbrous, too inadequate. The notion of function should take its place, just as in a factory an old machine is replaced by a machine of newer model and greater efficiency. Simple explanation, free of hypotheses, the bare statement of fact, is called description. The description of the actual is, therefore, the task of science. Mach takes the view (formulated for mechanics by Kirchhoff) that the end and aim of all the sciences is description, and, furthermore, the simplest possible description of the given. "Merely the relation of actual to actual is valuable and this is exhausted by description,"[2] as Mach says in reference to the explanation of magnetic attraction and repulsion, etc., by means of the theory of magnetic fluids. "The fluids added by thought have merely the properties that one must imagine them to possess for the purpose of explaining the actual."[3] The relations of attraction and repulsion should be regarded by science as actual; they should be described. In reference to such explanations in science, Mach says: "One might almost assert that the so-called descriptive sciences, commonly referred to with a tinge of condescension, have in point of scientific character surpassed the lately prevailing methods."[4]

[1] *A. d. E.*, p. 19. [2] *W.*, p. 437. [3] *Ibid.* [4] *P. V.*, p. 275.

The demand for description of the actual stands in a certain opposition to the demand for explanation. "How can the impression arise that explanation accomplishes more than description? If I show that an event *A* is of such a character as B, which is better known to me, then *A* becomes by this fact more familiar, quite as much so as if I had shown that *A* is a consequence of *B*, *C*, *D* (already known to me), or due to their combination. In the former case, one fact is substituted for another fact, one description for another description, better known to me. The event may by this means be made more apparent to me, a greater simplification may be gained; in its essence, however, no change takes place." [1]

Our feeling of intellectual satisfaction is more intense, a fact is clearer, in proportion to the simplicity of the description of the fact. "It is true, however, that the demands made upon simplicity are different for the specialist and for the tyro. A description by means of differential equations is enough for the former, while the latter may demand the gradual development from elementary laws." [2] The satisfying impression of clearness in descriptive explanation is due in large measure to the fact that the description of a fact by means of descriptions already well understood is the most economical.

The great universal laws of nature are descriptions of wide applicability and economy. In the natural sciences, where description is employed in a narrow sense, mathematical descriptions are available in lesser measure. Here economic thought employs the instrument of classification. Classification, also, is a frugal sort of description. For example, in its description of properties that are common to all the species of a family, classification employs the description only once. In this way, classification saves the repeated description of the single species.

Explanation and classification have this element in common: they do not repeat the description of a fact in treating other facts; they simply apply or transfer it. This is made possible by the similarity, *i. e.*, by the partial identity, of the facts. The search for similarities, for analogies, is, therefore, an important

[1] *W.*, p. 437. [2] *Ibid.*, p. 438.

element in every scientific inquiry. Discovered conformities make explanation and classification possible. They establish in general "the superiority of a scientific, methodical interpretation of a group of facts over an accidental, disordered interpretation, on the basis of the former's more frugal, economical use of mental forces." [1]

Similarities and analogies are discovered by comparison. "Memory is ever ready to offer for comparison such known facts as are similar to the new one; *i. e.*, such known facts as coincide with it in certain characteristics." [2] Consequently, comparison represents "the most important element in the inner life of science." [3] "The zoölogist sees in the wing-membranes of bats, fingers; he compares the bones of the cranium with vertebræ, the embryos of different organisms with one another, and the various stages of development within the same organism with one another. The geographer sees in Lake Garda a fjord." [4] "The philologist compares various languages and various forms within the same language." [5] The concept comes into being through comparative observation. "By means of the repeated application of such comparisons under manifold conditions, the inconstant characteristics, as measured by the constant and congruent marks, become so effaced that the latter take on a self-existent, abstract, notional significance, independent of every object and of every connection." [6]

Mach calls the description of facts by means of abstract notions direct description. The direct description of a mass of facts, although it represents an economic achievement, is formal when compared with indirect description. We have indirect description, "when we can say simply that the fact A now under consideration is like the known fact B not in one single mark, but in many or in all particulars." [7] Mach's meaning is explained by the following example: "The moon behaves like a heavy body in reference to the earth; light behaves like a wave-motion or electric vibration, the magnet like a body laden with gravitating fluids, etc." . . . "One readily sees that what we understand by a theory or theoretical idea, falls in the category of indirect description." [8]

[1] *Op. cit.*, p. 391. [2] *P. V.*, p. 266. [3] *Ibid.* [4] *Ibid.*
[5] *Ibid.* [6] *Ibid.*, p. 267. [7] *Ibid.*, pp. 267 f. [8] *Ibid.*, p. 268.

The superiority of theory consists in its economic capabilities. "The power of rapidly extending knowledge is what gives to theory its superiority over simple description." [1] This advantage, however, is accompanied by disadvantages. From the above will be seen the kinship between indirect description and explanation. The two conceptions to a great degree coincide. The dangers of explanation, its disadvantages when compared with simple description, constitute also the dark side of theoretical, indirect description. "Without contemptuously rejecting the serviceable help of theoretical ideas in science, yet as new facts become better known it might, in the light of the above considerations, seem not only advisable but even necessary to substitute direct description for the indirect; for the former contains nothing beyond the essential, and confines itself exclusively to the notional comprehension of the facts." [2]

The conception that the essence of theory consists in indirect description is intimately connected with the view according to which theories are images (Hertz's *Mechanik*). Theory completely grasps a series of phenomena in an image. It is, therefore, to be connected with the general imaging of facts in thought; theory, aiming at completion, brings new elements into the picture. The impulse towards such completion is given in our nature, without any addition on our part. "The impulse enriches to a certain extent the isolated fact. It lays in no way claim to infallibility, and there is no necessity that facts should coincide with it, and this is the weak side of theoretical ideas." [3] Of course, it is impossible that thought should give an absolutely complete copy of facts. New facts are discovered; better analogies, more far-reaching similarities, constantly change the details of the picture of facts which our thought outlines. "Ideas that mirror facts are not all of equal constancy. Always where we have a special interest in the imaging of facts, there will be an effort to support or strengthen the ideas of lesser constancy by those of greater constancy, or to substitute the latter for the former." [4]

Accordingly, the copies of facts in scientific thought are not

[1] *Op. cit.*, p. 269. [2] *Ibid.*, p. 275. [3] *A. d. E.*, p. 225. [4] *Ibid.*

immutable or eternally valid. As an example, one might cite here the notion of body or the more general notion of substance. As a picture or symbol of constant complexes, the notion once had a value, and it is still valuable for the needs of everyday life. But under the pressure of more exact experience, this notion has to be converted into the notion of a relatively constant complex of elements. All mental pictures and notions have to submit to similar "transformation and adaptation."[1] Facts can never be completely represented. Science carries on a constant process of transformation and of adaptation of ideas, symbols, and notions to facts, in such a way that incongruence is constantly diminished.

The biological conception of science is best illustrated in the investigation of the process of transformation. The points of view which Darwin established for the development of species are significant also for the evolution of ideas. "Ideas, like everything in nature, need time for germination, growth, and development; man with his thought is part of nature. Slowly, gradually, and with effort, one thought is transformed into another, as one animal species, in all probability, passes gradually over into another. Many ideas appear contemporaneously. Their struggle for existence is not different from that of the ichthyosaurus or the horse. Few persist long enough to spread over the various territories of knowledge, to divide, and so begin anew the struggle. As many an animal species, long since vanquished and belonging to a past age, continues to maintain existence in some remote region where it is not exposed to the attacks of its enemies, so we find ideas, long since obsolete, that continue to live in many a head. Whoever carefully observes his own mind will acknowledge that ideas keep up their struggle for existence as stiff-neckedly as do animals."[2] Mach cites as an example the transformation of the conception of a ray of light. First of all, the naïve observer regards a ray of light as a homogeneous straight line. Then, in the mind of Newton, it became the path of a projectile or an aggregate of paths of various and countless projec-

[1] *Cf.* the essay on "Transformation and Adaptation in Scientific Thought," *P. V.*, p. 243.

[2] *P. V.*, p. 75.

tiles. Then the conception had to adapt itself better to phenomena of interference. The ray was conceived in terms of periodicity, then in terms of vibrations. Finally, the conception was enlarged with reference to phenomena of polarization, and at last the ray of light lost entirely the character of a homogeneous straight line.[1]

Language exhibits a process of copying or reproducing facts, which has attained in the struggle for existence a brilliant development by means of its capacity for transformation and adaptation and by its use of economy. This mighty instrument of science has developed from insignificant beginnings, from animal language.[2] Scientific terminology exhibits only a more extended adaptation, an increased economy in language. "As far as the economy of written communication is concerned, one cannot doubt that science will some day realize the beautiful ancient dream of the philosophers regarding an international universal alphabet. The day is not so very remote. Numerical signs, signs of mathematical analysis, chemical symbols, musical notation, to which one might easily add a system of color symbols, and Brücke's phonetic alphabet, are significant beginnings."[3] As soon as advancing science has developed a clear notional system of adequate range, a corresponding symbolism will be developed, without any special decree, of a universal system of language-signs. So Mach believes that the old dream of the philosophers will come true, the dream of Leibniz, that we shall some day have a universal ideography. In this connection he cites the ideography prepared by the Italian G. Peano for the disciplines of mathematics.[4]

Mach's fondness for the notion of function, as above described, and its most far-reaching application, comes to clear view here. The elimination of the narrow notion of causality by the substitution of description in terms of dependences and processes with the help of functions, tends toward the realization of the ideal above portrayed.

The foregoing connected account of Mach's methodology might, perhaps, lead to a wrong conception of the character of his

[1] *Op. cit.*, p. 254. [2] *W.*, p. 407. [3] *P. V.*, p. 221. [4] *Ibid.*, p. 222.

thought. Positivism, in Mach's view, is a tendency rather than a system. The unsystematic garb of Mach's philosophy is presented in the form of philosophical comment scattered through his writings. All of his works are significant for his philosophy, the physical as well as the physiological-psychological writings.

The positivistic spirit pervades them all. If we look at the special sciences, this tendency of Mach's philosophy will, perhaps, become clearer. The conception of being and of the task of philosophy, with which the present article began, presupposes reference to the special sciences.

Let us begin with mathematics, more particularly with arithmetic. Counting is an orderly arrangement of signs,—primarily of the fingers, whose names gradually become such signs,— of members of an indefinite aggregate of similar things, which are representable as distinct from one another. "We count when we wish to show a distinct differentiation of similar things, *i. e.*, we give to each single thing a name, a sign."[1] Numbers are, therefore, fundamentally order-signs or order-numbers. Our numeral system is a system of order-signs, capable of indefinite expansion, and (by the introduction of fractions) capable of indefinite refinement.

The simplest arithmetical axioms are nothing more than very simple, immediately comprehensible experiences. "I conceive the propositions of mathematics as propositions of experience, even though they be derived from inner experience, and I long ago characterized mathematics as economically arranged numerical experience, prepared for use, whose aim is to substitute already performed numerical acts for direct numerical processes and for such processes as are frequently impossible."[2] Mach adopts an empiristic point of view, which Helmholtz held in similar form.[3]

In the same spirit Mach accepts the view of geometry developed by Gauss, Lobaczewski, and Riemann, the philosophical importance of which has been especially brought to light by the inquiries of Helmholtz. Given space is only one real space

[1] *W.*, p. 67. [2] *Ibid.*, p. 68.

[3] "Zählen und Messen," in the volume of Philosophical Essays, dedicated to Zeller.

amongst many conceivable spaces. The peculiarities of one space can be determined only on the basis of experience. These experiences become, in course of time, instinctive acts of knowledge, geometrical axioms. They confront us with such logical power as no consciously and intentionally derived knowledge can ever attain. Now, our mathematical space is determined by experience. It is not, however, for this reason identical with visual or tactual space. The points of visual space and those of mathematical space coincide, indeed, with one another, and a continuous passage from the point *a* in visual space to the point *b*, corresponds with the continuous passage between equivalent points in geometrical space. The origin of geometrical space is not purely a "matter of perception"; it is also a "matter of the understanding." "The space of the geometrician is a mental construction of threefold character, that has developed by means of manual and intellectual acts. Optical space (Hering's visual space) bears a complicated geometrical relation to the foregoing. . . . At all events, optical space has also a threefold character."[1] "The space of the geometrician exhibits at every point and in all directions the same characteristics, a fact which does not hold good of physiological space."[2] A series of experiences forces the understanding to substitute geometrical for physiological space. A mass of physical experiences are added to the system of space perceptions (visual and tactual) which are taken into account in the construction of geometrical space. "The fact that the geometrician regards his space as homogeneous in all points and in all directions, shows how far geometrical space transcends tactual and visual space."[3] "The basal propositions of geometry are actually derived from physical experiences, from the planning of lengths and angles, from the adjustment of rigid bodies to one another."[4] "Apart from the fact that images of space would never arise in consciousness without physical experience, we should in no wise be able to apply them to one another and test their congruence without such experience."[5] These physical experiences are not themselves imme-

[1] *A. d. E.*, p. 116. [2] *Ibid.* [3] *Ibid.*, pp. 232 f.
[4] *Ibid.*, p. 233. [5] *Ibid.*

diately necessary for the acquisition of geometrical truths, but merely the corresponding memories are necessary. "When we feel the necessity of representing an isosceles triangle as having equal angles at the base, this necessity rests upon the memory of cogent experiences." [1] Because the memory of experience is often adequate without the immediate presence of the experience itself, one is disposed to think that experience is something entirely apart from geometrical truth, that geometrical truth originates in "pure intuition." "But if a geometrical proposition were based on pure intuition, we should have no need to learn it. That discoveries are made by the activity of sheer geometrical imagination, as daily happens, only proves that the memory of experience can bring to consciousness moments which hitherto had remained unnoticed." [2] Meanwhile, the instruction of the young in geometry ought to appeal more frequently to experience.

Geometry is, therefore, to be regarded as an empirical science, as the "physics of space." [3] "The cogency of geometry does not rest on the fact that its doctrines are derived from a peculiar source of knowledge, but rather on the fact that its empirical material is easily accessible, has often been tested, and may at any moment be retested." [4] The province of space-experience is preëminently simple and of limited compass.[5]

The objection that the laws of geometry are completely valid, whereas geometrical notions are only imperfectly represented in the physical world, has no weight against the conception explained above. Geometry simplifies, idealizes, the empirically given for economic reasons. "If I revolve a crooked, thin, rigid wire about two of its points (ends) which are held fast, the others leave their position. The less crooked the wire is, the less these other points change their position. In so far as I *can or will entirely disregard* the crookedness, so far *can I disregard* the change in position. The straight wire, the straight line, is an *ideal*. In so far as I regard the ideal as *attained*, to this extent the straight line is determined by two points." [6] When geometry creates for itself constructions, formations that contain only

[1] *Loc. cit.* [2] *Ibid.*, p. 233. [3] *W.*, p. 546.
[4] *A. d. E.*, p. 233. [5] *Ibid.*, p. 234. [6] *W.*, p. 457.

what is put into them, then physical experience must teach us to what extent the objects of nature correspond with the creations of thought.

According to the foregoing, geometry can be regarded as a physical discipline. The construction of an ideal relation that approximates nature is not peculiarly a geometrical process, but a general and important physical means of scientific investigation. The perfect gas, that conforms to the Gay-Lussac-Mariotte law, is quite as much an ideal as is the straight line of the geometrician.[1] Idealized, elementary representation of the simplest processes of nature is the aim of physical investigation. Geometry lays claim to very few, simple provinces of experience; the whole of physics takes into consideration a somewhat wider territory. Because the experiences in question are so simple, and because of abstraction from all complicated processes, the description of this actual sphere is peculiarly 'exact.' "If physics with its methods accomplishes apparently more than the other sciences, we must, on the other hand, take into account that in a certain sense its tasks are far simpler." [2] The question: Does physics employ axioms? (P. Volkmann, *Physikal.-oekonom. Ges.*, Königsberg, 1894) is answered by Mach in the negative. Neither physics nor mathematics, properly speaking, has axioms. In Mach's treatise, *Die Mechanik in ihrer Entwickelung*, we have an extraordinarily brilliant investigation of physical, more particularly of mechanical, axioms. Unfortunately, it is impossible to take up here Mach's interesting criticism of the principles of Newton's mechanics, or the apparent axioms of statics. It is deserving of mention, however, that the theory of the conservation of energy, or as Mach is accustomed to call it, the "theory of the excluded *perpetuum mobile*, is merely a special phase of the law of causality [causal law in Mach's broad sense], which *results immediately from the view that phenomena are dependent on one another, a view issuing from every scientific investigation*, and which has absolutely nothing to do with the mechanical conception of nature." [3]

The passage cited shows Mach's attempt to rid himself of the

[1] *Op. cit.*, p. 457. [2] *Ibid.*, p. 439. [1] *E. d. A.*, p. 46.

mechanical interpretation of nature. Mechanical, more particularly, atomistic physics (and chemistry) are theoretical disciplines in the sense explained above. Physics should be, in the highest degree possible, a descriptive science. Nevertheless, Mach admits hypothetical ideas because of their value for clearness. Physics, however, has converted them into metaphysical realities, and has made of atoms substances in the "most naïve and crude way."[1]

If the atomic theory is merely a means of illustration, so under certain circumstances it may be practical to represent complicated atom-complexes in a space of more than three dimensions.[2] A series of relations must escape stereochemistry, if it confines itself to three-dimensional space.

The ideal of physics is a "complete, synoptical inventory of facts,"[3] from which all hypothetical, speculative elements are eliminated as superfluous. This ideal is to a certain extent attained in d'Alembert's (or Lagrange's) equations for dynamic facts, and in Fourier's equations for facts of heat transmission. When one speaks of Fourier's theory of the transmission of heat, one must remember that 'theory' is used here in a sense (viz., mathematical) very different from the ordinary one.

Physics and psychology do not concern themselves with entirely distinct spheres, as dualism supposes. Both of them have for their subject matter rather those products of analysis which Mach calls elements. Every science is a representation of the interconnections of certain elements by means of the elements $\alpha, \beta, \gamma \ldots$. (*cf.* above). "Physics (in the widest sense) arises through the representation of $A, B, C, \ldots$ in their interrelations; physiology or psychology of the senses through the representation of the relations of $A, B, C, \ldots K, L, M$; physiology through the representation of $K, L, M, \ldots$ in their relations to one another and to A, B, C. The representation of α, β, γ by means of other α, β, γ elements, leads to psychological science proper."[4]

Mach has enriched psychology by many beautiful experiments. His views differ not infrequently from those commonly held, as, *e. g.*, his views of space and time sensations, but we must pass over these suggestive and original investigations.

[1] *W.*, pp. 429 f.
[2] *E. d. A.*, p. 29.
[3] *W.*, p. 461.
[4] *A. d. E.*, pp. 219 f.

I should like to quote here a passage in explanation of Mach's view of the relation between the material and psychical, a passage that carries us back to the starting-point of this exposition of his monism of elements, or, somewhat inexactly expressed, his monism of sensation. "Whoever has at heart the unification of science into a whole, must look for some conception that he can employ in all provinces. If we analyze the *material world into elements*, which *at the same time* are also elements of the *psychical* world, and which, as such, are called sensations; further, if we regard as the exclusive task of science the investigation of the associations, interconnections, the reciprocal dependences of these homogeneous elements of all the provinces, then we may reasonably expect, on the basis of this conception, to build a consistent, monistic structure, and to rid ourselves of a distressing and confusing dualism. By regarding matter as the absolutely constant and immutable, one really destroys the connection between physics and psychology."[1]

[1] *Op. cit.*, p. 208.

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