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# Ernst Mach's "new theory of matter" and his definition of mass

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Mach's definition of mass-ratios was announced in his 1868 paper "Über die Definition der Masse," a treatment that remained essentially unchanged in the 1883 first edition of his *Mechanik in ihrer Entwicklung*. The definition became one of Mach's signal achievements; Einstein for example used it in his lectures on elementary mechanics at Zurich in 1909–1910 to call attention to the independence of gravitational and inertial mass from the chemical composition of bodies.[1]

Mach's definition of mass fits into a broader philosophical development in his thought during the 1860s having to do with the elimination of spatial properties from the fundamental concepts of physics. Although previous studies have looked into Mach's developing ideas on atomism,[2] this program's connection with the mass definition has eluded detection.

From 1863 to 1868, Mach was preparing what he called a "new theory of matter," with some important parallels to his work in physics. This theory developed in three major stages to be explored in this paper. The first consisted of finding fault with spatial "billiard ball" type atoms in two early papers on emission spectra and in his textbook of physics for medical students, the *Compendium der Physik für Mediciner* (1862–1863). At this time, Mach perceived difficulties

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[1] Martin Klein et al. Eds. *The Collected Papers of Albert Einstein* Volume 3 (Princeton, 1994), pp. 15–16, quoted in Jürgen Renn, "The Third Way to General Relativity: Einstein and Mach in Context" (*Max Planck Institut für Wissenschaftsgeschichte* preprints #9), p. 29. On the importance of the independence of gravitational and inertial mass from the chemical constitution of bodies see Einstein's *Autobiographical Notes* (LaSalle, IL: Open Court, 1992), pp. 60–63.

[2] Stephen Brush, "Mach and Atomism" Synthese 18 (1968): 192–215; Erwin Hiebert "Mach's Early Views on Atomism" in Cohen and Seeger (1970) eds. Ernst Mach: Physicist and Philosopher (Dordrecht: D. Reidel); Laurens Laudan "The Methodological Foundations of Mach's Anti-Atomism and their Historical Roots" in *Motion and Time Space and Matter* P.K. Machamer and R.G. Turnbull eds. Ohio State University Press, 1976.


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with contemporary atomic views, but had not yet proposed an alternative. In the second stage of his "Vorträge über Psychophysik" (1863), Mach put forward a "monadology" modeled to some degree on Johann Friedrich Herbart's theory of intelligible space and on Gustav Theodor Fechner's conception of an "inner side" of nature analogous to sensation. In the third stage, Mach eliminated these Herbartian monads and arrived at an early version of his theory of elements in a philosophical article "Über die Entwicklung der Raumvorstellungen" (1866). Here he laid out a program to eliminate spatial extension from physics in favor of intensive elements, which he compares to pressures. In "Über die Definition der Masse" (1868) Mach placed his concept of pressure "at the head" of mechanics, although he clothed his proposal in the language of spatial accelerations and ponderable matter. Thus, the mass-definition was not the completion of the "Raumvorstellungen" program by any means—that occurred only in Mach's late view of space—but it does seem to be a step along the way.

## 1. Atomism in the compendium

Mach used the textbook of his 1862 physics course for medical students as an organ to communicate his views on mechanical atomism:

> In the year 1862 I drew up a compendium of physics for medical men in which because I strove after a certain philosophical satisfaction, I carried out rigorously the mechanical atomic theory. This work first made me conscious of the insufficiency of this theory and this was clearly expressed in the preface and at the end of the book, where I spoke of a total reformation of our views on the foundations of physics (Mach, 1911, p. 86).

Mach approached atomism cautiously, laying out the bare features he considered indispensable: discreteness, impenetrability and obedience to the law of inertia (Mach, 1863a, pp. 11–12). His atoms exerted on one another a force of some nature falling off with the square of the distance. Atoms were also surrounded by cloud-like *Aetheratome* which repelled one another by forces of contact. Intra-atomic forces were not discussed. In fact, Mach was not even sure whether the atoms were chemically different among themselves or whether chemical differences emerged from varying combinations of identical atoms (Mach, 1863a, p. 15).

Mach appeared as a proponent of atomism in this work and gave several arguments as to why discrete particles were a more plausible view of matter than the competing theory of a material plenum. For example the conduction of heat through matter in wave motions implied that bodies consisted of strata of discrete particulate matter capable of vibration. (As Wolfram Swoboda has shown, Mach took over many of these pro-atomic arguments verbatim from G.T. Fechner's *Atomenlehre*, whose atomic conception was the model for Mach's *Compendium* treatment.[3]) Such was Mach's skill in rehearsing the conventional arguments and explanations of the theory that Erwin N. Hiebert writes:

> He had mastered the arguments of the atomic theory very well... although he later denounced the atomic theory vehemently and resolutely, he manifestly was not engaging in such polemics because of blindness or ignorance of the strengths and weaknesses of these conceptions (Hiebert, 1970, p. 95).

However Mach also complained that the proposed reduction of all physical processes to the atomic level had not gone through. He then considered the possibility that atomism had outlived its usefulness, and that this hypothesis would give way to a deeper "metaphysical" theory of matter:

> Here I have let the atomic theory step into the foreground everywhere, not indeed in the belief that it is the last and highest, and doesn't itself need further support, rather because it brings the appearances into a simple and visualizable [*anschaulichen*] association. The atomic theory, when one may express it thus, can be considered as a formula [*Formel*], which has already led to many results and will lead further to still more. In fact whatever kind of metaphysical opinion may arise in the future, the results won according to the atomic theory will be translatable into them, as one may express formulas in polar coordinates or in parallel coordinates (Mach, 1863a, p. vi)[4].

He inclined towards a new theory at the end of the book:

> We have now become acquainted with a series of physical phenomena and laws, which we, so far as it was possible by elementary means, have embraced under the point of view of mechanics and the atomic theory. A complete and strict tracing back of physical laws to a few exact principles is no longer possible today. The gaps are still too great. For heat and light phenomena we have at least found the most general contour of a theory and at least this contour rests on fixed foundations. We have not yet won so much insight into the phenomena of electricity and magnetism. In these areas there are empirical rules, to find one's way about in a large quantity of facts, rather than a real theory. However in this lies a stimulus to further research, and we hope that this will attain the goal, granted perhaps after a total reformulation of some of our fundamental physical views. At least the peculiarities of electrical and magnetic forces seem to point to the necessity of such a transformation (Mach, 1863a, pp. 271–272).

[3] Wolfram Swoboda "The Thought and Work of the Young Ernst Mach and the Antecedents to His Philosophy" doc. diss. University of Pittsburgh 1973, pp. 158–159. Sections published in German as "Physik, Physiologie und Psychophysik—Die Wurzeln von Ernst Machs Empiriokritizismus" in Rudolf Haller und Friedrich Stadler *Ernst Mach— Werk und Wirkung* (Wien: Verlag Hölder Pichler Tempsky, 1988), pp. 356–403.

[4] All translations that cite German language sources, including Mach's, are by me. Those that cite English sources follow the translator.

In the *Compendium* Mach's main objection to the mechanical view concerned Weber's law of electrical force between two moving, charged particles, which was also puzzling the world of physics at the time. The law said that, like gravity, the electrical force of attraction between two particles in motion was proportional to the charges and inversely proportional to the square of the distance. But it was *also* proportional to the velocity and the acceleration of the two charges. If the world consisted entirely of atoms attracting one another with central forces and otherwise obeying the law of inertia, such asymmetries between gravitational and electrical forces would not exist:

> This dependency of forces on velocity is still a riddle at present; it never happens when two bodies exert gravitational forces on one another for example (Mach, 1863a, p. 256).

In two physical papers published at this time, Mach proposed tentative steps towards replacing atomism with a deeper theory, without volunteering information on what this theory might be. The physical issue under discussion here was the bright-line emission spectra of gases. He said in 1872, reflecting back:

> My attempts to explain mechanically the spectra of the chemical elements and the divergence of the theory with experience strengthened my view that we must not represent to ourselves the chemical elements in a space of three dimensions. I did not, however, speak of this candidly before orthodox physicists. My notices in Schlömilch's Zeitschrift of 1862 and 1864 contained only an indication of it (Mach, 1911, p. 87).

In the earlier paper of the two, "Über die Spektra verschiedener Körper" (Mach, 1862), he suggested that the emission spectra of gases could be explained by the distances between vibrating atoms in a gas molecule.[5] When the atoms were shaken, oscillations in these distances produced the bright lines. There was no explicit skepticism about the existence of atoms in this article, but Mach limited himself in a tell-tale way to the observable facts of the spectral lines and the *distances*, without relying on the atoms as such. It was as if he proposed to consider distances without answering the question "distances between what and what?" Perhaps for this reason, among others, Mach's article struck physicists as "zu naturphilosophisch" (Hiebert, 1970, p. 99). Incidentally, Hugo Dingler writes that, due to his philosophical views, Mach was regarded by his physicist colleagues at this time as a hard-to-understand loner (*ein schwer verständlicher Eigenbrödler*) (Dingler, 1924, pp. 24–25).

[5] Perhaps Mach attributed the emission spectra to "molecules" and not "atoms" because he thought that atoms entered into combinations in molecules which in turn led to the chemical and spectral differences. The distances between these atoms *in a gas molecule* are the cause of the spectra and not the internal structure of the atoms themselves. We now know that the intraatomic structure is the cause of the unique spectrum for each chemical element.

Mach followed this up with an 1864 article "Vorläufige Bemerkungen über das Licht glühender Gase"(Mach, 1864) in which he said to his critics:

> I must remind you that I by no means consider the atomic theory as something established in itself, but rather as a useful temporary empirical formula; as a kind of *regula falsi* to be used in drawing closer to the truth (Hiebert, 1970, p. 100).

He then repeated his suggestion from the *Compendium* that the results of the atomic theory could be "translated into a different "metaphysical" view of matter, which henceforth would prove to be more cogent." And now he explicitly declared his support for an alternative:

> I personally am now already very much inclined toward such a translation. And I shortly hope to show that my remarks by no means hang together so necessarily with theories, but can be understood very readily as an expression of the facts (Hiebert, 1970, p. 100).

This alternative view was influenced by Mach's physical interests. As a student, he wanted to investigate the constitution of matter and saw the phenomena of light and heat radiation emanating from bodies as the key to uncovering it. For example, on September 26, 1860 Mach sent a letter to G.R. Kirchhoff describing a method for isolating the spectrum of a distant phosphorescing body from the spectrum of its reflected light by using a polarizing filter, which got a gruff answer from the Master, and not the promise of financial support Mach anticipated (Thiele, 1978, p. 32).

*Unlike* most physicists, Mach came to see the wavelike transmissions of energy not merely as a means of inferring the constitution of matter, but perhaps as the underlying constitution itself. In his notebook of 1874, for example, he wrote that he considered the passage of matter through space to be a wave, or a propagating "potential difference":

> The movement of matter is a wave. A difference that progresses forward…Light as periodic potential.[6]

In itself a wave theory of matter does not imply a deeper level of description under matter, the matter itself could move in wave formations without being comprised of them. But this is not the only indication Mach favored a sub-spatial theory involving waves as matter. In Mach's (1866) paper "Über die Entwicklung der Raumvorstellungen" he wrote that he considered light a "proto-chemical" separation and combination, because light interacted chemically with bodies and solutions (Mach, 1866, p. 232). His idea here was not only to compare light to a chemical process, but also to conceptualize chemical processes within matter as similar to light waves. Further, he pointed out that chemical changes in matter might not be due to the motion of bodies lying in space "next to one another" but that the phenomena of motion might really be qualitative "chemical" changes (Mach, 1866, p. 232). Had he not been frustrated in his attempt to study physics with Franz Neumann in

[6] Die Fortbewegung d. Materie ist eine Welle. Eine Differenz die fortschreitet…Das Licht als periodische Potential. Notebook dated 18 September 1874: (NL 174/02/05).

Königsberg (Blackmore, 1992, p. 24) it is likely that he would have pursued these interests in a physical framework. Instead he pursued his studies in sense physiology and developed a kind of natural philosophy.

## 2. Herbart's Metaphysik

A very significant influence on Mach's theory of matter and space came via the German philosopher Johann Friedrich Herbart (1776–1841), a student of Fichte who rejected idealism and returned to a realistic Leibnizean metaphysics. Mach said that in 1862 he was studying Herbart's "intelligible space," a construction in the philosopher's *Allgemeine Metaphysik*:

> I was busied at the same time with psychophysics and with Herbart's works and so I became convinced that the intuition of space is bound up with the organization of the senses, and, consequently that we are not justified in ascribing spatial properties to things which are not perceived with the senses…At the same time the quite arbitrary and, on this account, faulty limitation of the number of dimensions in Herbart's derivation of "intelligible" space struck me. By that, now, it became clear to me that, for the understanding, relations like those of space, and of any number of dimensions, are thinkable (Mach, 1911, pp. 86–87).

Mach's philosophical views of this time explain better why he thought it necessary to reject an atomic theory that fitted the observable evidence quite well across all of physics. Mach had read Kant's *Prolegomena* as a boy and may have been impressed by the clearly stated injunction not to extend spatial properties to the world independent of perception.[7] If Kant did not carry the point far enough, Herbart certainly did by presenting a psychological theory of spatial manifolds firmly separated from the intelligible space of his metaphysics, although both were considered different orderings of the same sensational content (Herbart, 1829, p. 21), as in Mach's later theory of neutral monism, presented in the *Analysis of Sensations*.

In Herbart's view, space resulted from a level beneath, occupied by spaceless monads called *Wesen* (beings, essences) and their qualities.[8] While the Wesen were simples they could be distinguished by their different qualities. Herbart described the *Wesen* as "pressing" one another through their qualities, as if these were actually analogous to forces. These confined and confining pressures, overcoming and giving way to one another, ultimately generated quasi-spatial waves that moved abstractly as the underlying intensities changed. It thus seems likely that when Mach referred to matter in his notebook as a wave, passing over differences of potential level, he was drawing on Herbart's construction of space.

Although Herbart placed great stress on the existence of his Wesen, they struck Mach as dispensable in the construction of space. The stability of the Wesen

[7] *Prolegomena to any Future Metaphysics*. Carus and Ellington trans. (Indianapolis: Hackett, 1977) Part I, Remark III.

[8] A reconstruction of Herbart's ideas in relation to Kant and Riemann can be found in Banks (2003b).

depended on the equilibration of the pressures exerted by their qualities; hence it was really only the intensive relations of qualities that mattered because these determined both the stable forms of objects and the tracing out of extended regions. Only where the qualities instantaneously cancelled one another in strength were there Wesen to be found:

> We could say through a sensory comparison what they [the Wesen] do. They *press* one another. For in the world of the senses we find resistance in pressure, where nothing gives way although each is supposed to move. Pressure is rest, through reciprocal endurance against another. But sensory comparison is dangerous here. We are not talking about spatial relations…Here we are only talking about a change in quality that each should suffer from the other, however against which it conserves itself as that which it is. Disturbance should result; self-conservation cancels the disturbance in such a way that it does not occur at all (Herbart, 1829, Section 234, p. 103).

Mach's philosophical critique of the *Allgemeine Metaphysik* is now seen as a main influence leading to his own theory of elements (Laass, 1991, p. 213). Moreover, when we bring in Mach's contemporary work in science, the philosophical move of eliminating the *Wesen* for the underlying qualities paralleled exactly Mach's proposal to ignore atoms and concentrate on their potential to emit spectral lines. In fact these developments occurred at roughly the same time, from 1862 to 1863.

Mach also rejected Herbart's claim that space was limited to a three-dimensional manifold, a criticism that may also have occurred to two other prominent readers of Herbart: Bernhard Riemann and Hermann Grassmann (see Banks, 2003b). According to Mach, Herbart had artificially limited the generating process of space to three dimensions. Mach pointed out that this limit was psychological, not logical or even physical, and hoisted Herbart with his own petard. In Mach's *Conservation of Energy* of 1872 and in his 1871 article "Über die physikalische Bedeutung der Gesetze der Symmetrie" he parlayed this critique of Herbart into an argument against three-dimensional molecules, writing that the "physiologically based" assumption of 3-space for physics artificially limited the number of possible distances between atoms. This error arose by assuming those atoms were already embedded in 3-space, rather than letting space arise from their physical interactions (Mach, 1911, pp. 51–54). In response, Mach drew up a table of the number of atoms in a molecule and the distances between them in three dimensions. It could then be expected that given a certain number of distances the rest would be determined. For example, in a molecule of five atoms in 3-space, nine distances determine the tenth. But distances between atoms were an empirical datum. Thus if those distances were given and the rest were not determined, Mach could argue that space in the small was not three-dimensional (Weinberg, 1937). Indeed Mach even suggested that spatial distances be replaced with a physical property of pairs of $n$ parts of a molecule:

> The heat of combustion generated by a combination gives a clearer picture than any pictorial representation. If, then, it were possible, in any molecule composed

> of $n$ parts, to determine the $n(n-1)/1.2$ heats of combination of every two parts, the nature of the combination would be characterized thereby (Mach, 1911, pp. 53–54)[9].

Moreover, since Mach attributed the spectral lines to oscillations in the $n$ parts of a molecule, he implied that he had found more lines due to independent directions of oscillation than the possible distances between atoms suspended in three-dimensional space. Thus extending the physiological property of three dimensionality to physical space could be exposed as a metaphysical error.[10] That threat, although it is only expressed fully in the "Symmetrie" paper and the *Conservation of Energy*, seemed already to hang over Mach's earlier articles on emission spectra addressed to physicists.

Mach made good on his threat at the end of his "Symmetrie" paper, applying his physiological critique to the "billiard ball" atomic theory, of course before he had evidence of atomic structure, and said "those theories which derive all phenomena from the motion and equilibrium of the smallest parts, the so-called molecular theories, are already set tottering due to the theory of the senses and of space, and one may say that their days are numbered" (Mach, 1871b, p. 146). He then proceeded to show that a space of one dimension was given for the sensations of sound, a Tonraum or tone space analogous to three-space but possessing fewer dimensions. This space had certain properties which could be discovered by experiment, for example melodies could be recognizably preserved through a transformation of major into minor keys, just as certain figures in visual space retained their properties through transformations. Mach then imagined creatures in tone-space trying to formulate a physical theory by imagining that the world, too, is a one-dimensional manifold:

> If someone who could only hear wanted to try and develop a world-view in his linear space, he would come up considerably short, in that his space is not equal to the many-sidedness of the real relationship. It is no more justified when we think we can press the entire world, even the unseen parts, into the space of our eye. But all molecular theories fall under this head. Yet we possess a sense which, in respect to the many-sidedness of the relations it is capable of contemplating, is richer than any other. It is our reason. It stands above our senses. It alone is in the position to establish a lasting and complete world view. The mechanical world view has achieved enormously much since Galilei. But it must make way for a freer perspective (Mach, 1871b, p. 147; cf. Mach, 1911, p. 50).

[9] Mach had obviously changed his mind to allow that individual atoms were chemically different, and not identical corpuscles, since each part of a molecule was allowed to have a different heat of combination, a different chemical property.

[10] In connection with his late views of space as a Riemannian chemical manifold, Mach reversed himself later and decided that space was three-dimensional after all (Mach, 1911, p. 94; Mach, 1960, pp. 103–104). However, behind this three dimensionality were still physically determined facts: three essentially different "chemical directions" (Mach, 1959, p. 339) and a physical dependence of these directions on one another (Mach, 1987, pp. 506–508). See the last section of this paper.

He then adds in a footnote:

> It follows from this that the dependence of natural phenomena be expressed through relations of number, not spatially or temporally (Mach, 1871b, p. 147n).

With the benefit of historical hindsight, we know that in his *Conservation of Energy*, Mach would go on to develop his own element and function *Elementenlehre* for representing physical phenomena without the inclusion of spatial and temporal properties in the fundamental description. Even Mach's conception of functional dependence did not smuggle in either space or time, as he later insisted:

> I think I must add, and have already added in another publication, that the express drawing of space and time into consideration in the law of causality is at least superfluous. Since we only recognize what we call time and space by certain phenomena, spatial and temporal determinations are only determinations by means of other phenomena (Mach, 1911, p. 60).

Mach demonstrated considerable mental flexibility in contemplating aspatial and atemporal functional connections between events, criticizing the idea, which he attributed to Wilhelm Wundt, that locomotion, or change of place, was the only kind of change admissible in physical science because it was the only transformation in which a thing remained identical to itself. Qualitative changes were equally admissible, Mach maintained, if "a thing is destroyed in one place and in another an exactly similar thing is created" (Mach, 1911, p. 50).[11]

With many of these critical points no doubt already in his mind, Mach rapidly began outlining his alternative view of matter. In his 1863 "Vorträge über Psychophysik," for example, he declared that he wished to eliminate the hypothesis of "spatial atoms" (Mach, 1911, p. 87; Mach, 1863b, p. 364)[12] in favor of *Wesen und Kräften*, i.e. a Herbartian manifold, possibly multidimensional in nature. He expressed his opinion that space resulted "very probably from a mediate interaction of a plurality of Wesen" (Mach, 1863b, p. 364) and then cited his paper on gaseous spectra as a possible means of verifying this construction:

> In… (*Schömilch's Zeitschrift für Mathematik* VII. Jahrg. III. Heft) I have shown how one may deduce the nature and constitution of gas molecules from the results of the latest spectral investigations of light from glowing gasses. From the oscillations that shake a system, one may draw conclusions about its constitution…If we wish to mix in as few hypotheses as possible then we must say that physics leads always and everywhere to the idea that it is a finite number of Wesen and forces that are active in the phenomena. (Mach, 1863b, p. 364).

[11] Barry Smith and Kevin Mulligan call attention to Mach's advanced notion of functional dependence, which they call "non-causal," in "Mach and Ehrenfels" in B. Smith ed. *Foundations of Gestalt Theory* (Munich and Vienna: Philosophia, 1988) pp. 124–57. I think it is better to say 'causal but aspatial and atemporal' since Mach's notion of functional connections is prior to spatial and temporal properties usually included in the notion of cause (even today!)

[12] (Mach, 1911) p. 87: "In my lectures on psychophysics I already stated clearly that we are not justified in thinking of atoms spatially."

It may seem strange that Mach was drawing his ideas of matter and space from a German metaphysician given Mach's supposed antipathy toward metaphysics. But even in the *Compendium* Mach had acknowledged the role of philosophy in natural science, alerting his readers that philosophical work continuous with science might be necessary to see physical problems from another side, once one had a command of the physics:

> Where philosophical questions are touched on, which was not completely to be avoided, I restricted myself to the ground of natural science, in part because this standpoint has a certain authority, in part also in the conviction that one must have tasted of this outlook for a time to feel the necessity that the investigation be pursued from another side; some never get this far.

Mach's later critique of metaphysics was never an attack on unobservables per se (Mach, 1960, pp. 587–588). In fact, I have found considerable evidence that Mach admitted a whole class of unobservable elements himself to fill out his own world view.[13] As Mach made clear in his "Symmetrie" paper, (Mach, 1871b) the "antimetaphysical critique" was really the objection of the sense physiologist he was, attempting to separate the pure conceptual content of physics (given by the elements and functions) from the residue of physiological properties that helped human beings (physicists included) to visualize physical events.[14] A typical "metaphysical" error, as explained by Mach, is committed by the investigator who finds out that images of objects on the retina are oriented "upside down" and wonders how on earth the person can see them "right-side up" (Mach, 1959, pp. 38–39). Here the problem is that the spatial orientations "up" and "down" have different meanings in the manifold of sight from the meaning employed when speaking of the physical orientation of objects outside. Mach opposed the anthropomorphic tendency in physics to ascribe sensory properties to objects outside the sensory manifolds in question.[15] Thus Mach's ideas do *not* translate well as 20th century concerns about the relations between observable and unobservable (or theoretical), terms or objects.

[13] In my forthcoming book (Banks, 2003a) I have assembled the evidence that Mach believed unsensed sensations existed, similar to Russell's sensibilia, and even, surprisingly, mind-independent sensations in inanimate matter: "Die Empfindung ist eine allgemeine Eigenschaft der Materie…Die Materie empfindet" ("Uber einige Hauptfragen der Naturwissenschaft" Vortrag Sommer 1872 (NL 174/1/003) reproduced in Haller and Stadler, 1988, cf. Mach an Friedrich Adler Jan. 23 1910 in Blackmore and Hentschel, 1985).

[14] It is true of course that Mach regarded the physical and the psychical as two different orderings of the same content (Mach, 1911, p. 91; Mach, 1959, p. 16; Banks, 2003a). But the orderings of the natural realm may be more general than those elements presented to sensibility in manifolds of physiological space and time.

[15] One might object: why does Mach feel justified in asserting that the nature of physical elements is akin to what we know as our sensations? That is, why does he permit such an analogy concerning the content of experience (sensation) but not the form (space and time)? See Banks (2003a).

## 3. The "Vorträge über Psychophysik"

Mach's lectures on psychophysics were the next step in his development of a theory of matter, in which still another problem was under attack. These lectures were addressed to Viennese doctors interested in the latest sense physiology of Fechner, Wundt and Helmholtz. Mach used the occasion to call attention to problems in the unity of science raised by the new psychology. For instance, If psychology employed basic units of sensation (as Mach, Wundt and Fechner thought it did) how could these ever be reduced to the matter-and-motion of quality-less atoms? Once again Mach found himself in conflict with the mechanical philosophy, which he felt could not be the unified science he was seeking. His solution was to propose his own psychophysical monadology, a forerunner to his theory of elements and a theory capable of accommodating the sensations of psychology and the inanimate matter of physics in one system—but only by doing away with atoms in favor of Herbartian *Wesen* and their pressure-like qualities. Instead of eliminating qualities in favor of a swarm of mechanical atoms, Mach allowed that even inanimate matter was made up of monadic qualities of its own, an "inner side" of Nature:

> How do we have to think of these Atoms? Colored, lighted, sounding, hard?
> These are sensory properties which atoms only have in community with one another, for all physical phenomena result from a plurality of atoms. We cannot attribute these properties to one atom alone. We cannot even think of atoms as spatially extended. For as we have seen space is nothing original and results very probably from a mediate interaction of a plurality of Wesen. Physicists, too, have already felt the difficulty of imagining atoms concretely, and for that reason some consider the atoms as mere centers of force. But a center of force for itself is actually nothing. And what does it mean anyway to say one center of force acts on another? Let us simply admit it! We cannot reasonably succeed in giving any kind of outside to atoms; should we think anything at all we must attribute an inner side to it, an inner side in some respects analogous to our own soul (Mach, 1863a, p. 364).

By enlivening matter with qualities similar to sensations, Mach was following his mentor of those years, G.T. Fechner. In his *Psychophysik*, Fechner had proposed a theory of mind in which mental properties were simply the appearance of the brain to itself, a monadic perspective available only to the possessor.[16] Physical properties were the appearance of the brain and other physical objects to one another, "external" properties like extension, which could be shared by many minds at once. In Fechner's famous example, a circle cannot be convex and concave at the same time, and similarly no one can both possess the sensations of his brain and observe them from the outside simultaneously (since the sensation exists uniquely but once and for itself alone). Fechner's

[16] G.T. Fechner. *Elements of Psychophysics*. Helmut A. Adler, trans. (NY: Holt, Reinhart and Winston, 1965), pp. 1–2.

cleverness lay in playing off identity and dual-aspect theories against one another: invincible duality proves identity, since nothing could be so mutually exclusive in its aspects as interior and exterior aspects of the same thing; but, by the same token, identity demands a duality of aspects since, for him, nothing could both *be* itself and enter into relations with itself from the outside. As part of his very comprehensive study of Fechner's philosophy, Michael Heidelberger relates that Fechner's criterion of identity of an object is this functional variation of the thing's inner "mental" aspects with its outer physical aspects all combined as one object, *not* the further inherence of those aspects in some Spinozist substance or *tertium quid* (a point I think Mach failed to appreciate) (Heidelberger, 1993, pp. 123–142).

Mach's *Monadenlehre* in the "Vorträge" was, as he says there, an attempt to find a Fechnerian solution to the problem of inanimate matter and its relation to mind; he also says he is of Fechner's opinion that all of nature possesses an inner side analogous to mind (Mach, 1863b, pp. 363–364). Mach's critics among his physicist colleagues had indeed smelled out in his earlier paper a *naturphilosophische* construction of matter brewing behind the positivistic proposal to examine only the "observable phenomena" of the spectral lines. Meanwhile Fechner himself could neither accept what he called Mach's "*abweichender*" (divergent) standpoint, nor what he perceived to be Mach's continuing attachment to Herbartian psychology.[17] As a coda to their congenial correspondence of the early 1860s, Fechner politely refused the dedication of an early version of the *Analysis of Sensations* written before 1864. It must have been quite a blow. Mach confided at 73 years of age, and long after he had rejected Fechner's psychophysical law and measure of sensations (Mach, 1959, p. 81n; Mach, 1986, pp. 50–51),[18] that Fechner's rejection still "pursued him in his dreams" (Heidelberger, 1993, p. 208).

## 4. Mach's "Raumvorstellungen" Article

Further constructive steps towards Mach's "new theory of matter" were presented in a philosophical article "Bemerkungen über die Entwicklung der Raumvorstellungen" (Mach, 1866). Here aspects of Mach's mature *Elementenlehre* made their first appearance. Objects were now completely broken up into qualities and their functional relations, as in the later *Analyse der Empfindungen*. On the physical side, Mach began his all-out attack by analyzing spatially extended matter into constituent pressures:

[17] Fechner calls Mach's standpoint *abweichend* (divergent) in a letter to Mach of December 11, 1865; the refusal of the dedication is found in Fechner's letter to Mach of April 18, 1864. The originals, written in deutsche Schrift, are at Deutsches Museum and are transcribed in Joachim Thiele's *Wissenschaftliche Kommunikation: Die Korrespondenz Ernst Machs* (Kastellaun: A Henn Verlag, 1978). Thiele (p. 41) speculates that Mach's first draft of the *Analysis* held a theory of monads such as presented in the "Vorträge."

[18] Mach's specific objections to Fechner's scaling of sensation-intensity to the logarithm of the stimulus were contained in his lecture "Über einige Hauptfragen der Physik Sommer" 1872 (NL 174/1/003).

> It cannot be my intention here to criticize our conceptions of matter whose insufficiency is, indeed, generally felt. I will merely make my thoughts clear. Under [the rubric of] matter, let us imagine, then, a something in which different states can occur, say for simplicity a pressure in it, which can become greater or smaller (Mach, 1866, reprinted in Mach, 1911, pp. 88–89).

He then attacked the representation of spatial distances, claiming that the law of force in nature holding between pieces of matter need not be thought of spatially, but rather even the distances themselves in the law of force could be derived in reverse from the intensity of the forces:

> Physics has long been busied in expressing the mutual action, the mutual attraction (opposite accelerations, opposite pressures) of two material particles as a function of their distance from each other—therefore of a spatial relation. Forces are functions of the distance. But now, the spatial relations of material particles can, indeed, only be recognized by the forces which they exert on each other. Physics, then, does not strive, in the first place, after the discovery of the fundamental relations of the various pieces of matter, but after the derivation of relations from other, already given, ones. Now it seems to me that the fundamental law of force in nature need not contain the spatial relations of the pieces of matter, but must only state a dependence between the states of the pieces of matter (Mach, 1866, reprinted in Mach, 1911, pp. 88–89).

This is the passage that is relevant to the mass definition. I see Mach as proposing that both pressures of contact (in collisions due ultimately to inertial mass and the square of the velocity) and pressures due to fundamental forces (due to gravitational attraction and proportional both to mass and to distances between attractive masses) could both be set down as the same kind of "pressure," [*Druck*]. It would then be possible from a table of such pressures to specialize those that increase (decrease) due to the greatness of the masses, and those that increase (decrease) as a function of the varying distances between constant masses.

Mach concludes the "Raumvorstellungen" article by imagining a physics in which all of the pressure-states could be set down as mutually dependent functions of one another, a completely abstract dependence as the foundation for the forms of space, time and ponderable body (terms Mach claimed he could eliminate):

> If the positions in space of the material parts of the whole universe and their forces as functions of these positions were once known, mechanics could give their motions completely,[19] that is to say, it could make all the positions discoverable at any time, or put down all positions as functions of time.
>
> But, what does time mean when we consider the universe? This or that "is a function of time" means that it depends on the position of the swaying pendulum, on the position of the rotating earth, and so on. Thus "all positions are functions of time" means, for the universe that all positions depend upon one another.

[19] [Translator P.E.B. Jourdain's note: For this purpose it would be necessary also to know the velocities of the various parts at that instant.]

> But since the positions in space of the material parts can be recognized only by their states, we can also say that all the states of the material universe *depend upon one another*.
>
> The physical space which I have in mind—and which at the same time contains time in itself—is thus nothing other than *dependence of phenomena on one another*. A complete physics which would know this fundamental dependence, would have no more need of special considerations of space and time, for the latter considerations would be included in the former knowledge (Mach, 1866, reprinted in Mach, 1911, pp. 88–89).

Thus all of the relations between seemingly heterogeneous conceptions of matter moving across spaces can be cashed out in terms of pressures and functions of these, in an ambitious philosophical-physical program.[20]

## 5. "Uber die Definition der Masse" (1868)

Now turning to the mass definition itself, the original paper "Über die Definition der Masse" appeared in *Carls Repertorium für physikalische Technik* along with two other notes in 1868. It had been submitted 1 year earlier and rejected by Poggendorf, the editor of the *Annalen der Physik* (Mach, 1911, p. 80). Mach claimed to have developed his physical ideas much earlier in his lectures and private historical studies, but had avoided presenting them to the stiff-necked community of physicists. He said in 1872, reflecting back:

> …these thoughts, which, as the notes and quotations from my earlier writings show, are not of very recent date, but which I have held since 1862, were not suited for discussion with my colleagues—I, at least, soon tired of such discussions. With the exception of some short notices written on the occasion of other works and in journals little read by physicists, but which may suffice to prove my independence, I have published nothing about these thoughts (Mach, 1911, p. 16).

Newton was the first to separate mass from weight and to determine by experiment that a body's mass was independent of its chemical constitution (Mach, 1960, pp. 235–238). For Newton, if we attempt to accelerate a heavy object it offers resistance, on earth or even in empty space with no other bodies in the immediate neighborhood. Mass and weight are proportional because experimentally objects twice as heavy offer twice as much resistance to acceleration. But the same mass may have many different weights on other planets, because of their different gravitational pulls, while the inertial mass remains the same. Indeed the proportionality between inertial and gravitational mass explains why objects of different masses fall at the same rate in a gravitational field; the extra pull of gravity on the heavier masses is

[20] It is necessary to know the momenta of the masses as well, but there is no reason why these too could not be conceptualized by pressures of collisions or resistances to changes in angular momentum.

offset by their increased inertial resistance to acceleration. In addition to these experimental criteria, Newton gave a completely independent definition of mass as the "quantity of matter" contained in a body.

Mach's (1868) definition of mass was not a sharpening or refinement of the stated Newtonian "quantity of matter" but a new concept of *mass ratios* ($m_\alpha/m_\beta$). Individual mass-values (a pair of a body and a number for example) did not exist for Mach; even the individual terms of mass ratios, like $m_\alpha$, are themselves ratios.[21] This fact must be kept firmly in mind. Mass-values were the result of a system of chained comparisons between one ratio and another and with a standard ratio, which could be verified experimentally. Thus Mach was really defining values for a *system* of masses. To say that mass does not vary in a certain event means not that a certain lump of matter is the same as it was before, but that a systematic relationship between all of the ratios is upheld. Consequently, there are properties of Newtonian quantity of matter that mass ratios lack. For example, there is no mass ratio for the entire universe. Moreover, because of the close relationship between Machian mass ratios and kinetic energy—manifested most directly if the comparisons between masses are made by a system of momentum-conserving collisions—they can be included within the special theory of relativity with only a few modifications (Yourgrau & Van der Merwe, 1968, pp. 247–248).

Mach's definition appealed to an experimental procedure. We let two bodies ($K_\alpha, K_\beta$) act on one another by whatever forces they mutually exert, electrical, magnetic and gravitational attractions, and forces of contact, such as the vis viva communicated in collisions. The forces produced by the one body ($K_\alpha$) cause accelerations in the other ($K_\beta$), which must be accepted as a fact of experience. It is not evident a priori that forces or pressures must manifest themselves as accelerations in the direction of the forces.

If we call the first body $K_\alpha$ and the other $K_\beta$ then $K_\alpha$ acquires an acceleration $\varphi_{\alpha\beta}$. Meanwhile $K_\beta$ acquires an acceleration $\varphi_{\beta\alpha}$ (which we read as "the acceleration of $K_\beta$ due to $K_\alpha$"). Because the forces both bodies exert on one another are equal by action and reaction, we write:

$$F_{\alpha\beta} = -F_{\beta\alpha} \tag{1}$$

$$m_\alpha \varphi_{\alpha\beta} = -m_\beta \varphi_{\beta\alpha} \tag{2}$$

or

$$m_\alpha/m_\beta = -\varphi_{\beta\alpha}/\varphi_{\alpha\beta}. \tag{3}$$

Now it is often assumed in the literature that Mach intended for his definition to reduce "unobservable" pressures or forces to grossly observable spatial bodies and accelerated motions (for example Russell, 1954, p. 19). That is an understandable reading, of course, since it looks (in (3)) as if the mass-ratios ($m_\alpha/m_\beta$) are being *defined* in terms of the ratio of their accelerations ($-\varphi_{\beta\alpha}/\varphi_{\alpha\beta}$). But since Mach's definition ultimately refers to a system of reciprocally determining ratios it is not a

[21] Arnold Koslow used just this point to argue that the non-existence of unique individual mass-values does not imply the same for mass-ratios (Koslow, 1968, p. 226).

classical definition, made up of a *definiendum* and a *definiens*. In fact, Mach believed that forces (or pressures) *were* the readily demonstrable data of experience. In this respect (as Koslow notes 1968, p. 222) Mach espoused a belief in the reality of forces, descending from Newton himself. Mach even said in his criticism of Heinrich Hertz's *Mechanik* (which really *did* try to dispense with forces):

> To characterize forces as being frequently "empty running wheels" as being frequently not demonstrable to the senses, can scarcely be permissible. In any event, "forces" are decidedly in the advantage on this score, as compared with "hidden masses" and "hidden motions." In the case of a piece of iron lying at rest on a table, both the forces in equilibrium, the weight of the iron and the elasticity of the table are very easily demonstrable (Mach, 1960, p. 319).

As an empiricist Mach was acutely aware that fundamental forces of various natures manifest themselves as pressures. In nature we can immediately demonstrate pushes and pulls by finding other standard pushes and pulls to oppose them. We can still feel the pressure in the tension each offers the other without producing motion. For example, without producing motion, we can slip a platform beneath the iron to replace the pressure offered by the table, and we can switch the iron for an equivalent test weight. These motionless tensions are experimentally demonstrable facts and show that pressures can be obtained due to the relative positions and the greatness of the masses.

Mach stated in his notebook, that pressure should be measured with pressure [*Druck wie d. Druck messen*].[22] The experimental proposition that masses induce pressures in one another would be verified by impressing test pressures. Since *Druck* is experimentally fundamental, we need not even know yet that it can be further analyzed into the product of mass and acceleration.

In the "Definition of Mass" article itself we indeed find that pressures play the primary role, just as they did in the "Raumvorstellungen" paper. But what Mach means by pressure (Druck) is the force due to mass (mg) and not force per unit area, which is the contemporary meaning. Mach began by attacking the logical circle in the concepts of mass, acceleration and *pressure*:[23]

> $m = p/g$, $p = mg$. This is either a very repugnant circle or we are required to conceive of force as "pressure" (Mach, 1868, p. 356).

The reason there was a vicious circle, as he later observed in his *Mechanics*, was that mass and pressure are always conjoined: wherever we have masses they attract one another and induce mutually opposing pressures, and all pressures considered in mechanics proper are due to the presence of masses (attracting or producing acceleration or negatively resisting acceleration). Nowhere do we find a pressure or a

[22] Notebook dated 4 May 1880 (NL 174/2/16).

[23] Not mass = density · volume, density = mass/volume, which Mach first referred to as a "pseudo-definition" in the *Mechanics* (Mach, 1960, p. 300). ($M$ = mass, $p$ = pressure, force due to mass, $g$ = acceleration of gravity.)

mass alone. Mach thus proposed to put pressure "at the head of" mechanics along with action and reaction:

> In that principle of Newton, which is customarily placed at the head of mechanics and which runs "Actioni contrarium semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi," the "actio" is again a pressure or the principle is quite unintelligible unless we already possess the conceptions of force and mass. But pressure looks very strange at the head of the quite phoronomical mechanics of today. However this can be avoided (Mach, 1868, p. 357).

Mach "avoided the problem" by making the fundamental law of mechanics a *dynamical* principle: two qualitatively indistinguishable bodies always induce in one another equal and opposite accelerations (*Beschleunigungen*):

> *Proposition of Experience*: Bodies placed opposite one another communicate to each other accelerations in opposite senses in the direction of their line of junction (The law of inertia is already thereby included (Mach, 1868, p. 359).[24]

This proposition is really tantamount to assuming Newton's law of universal gravitation, because it includes the knowledge that all masses attract and accelerate towards one another. (By contrast Newton may have seemed to Mach to have defined force as a deviation from the rectilinear motions specified in his law of inertia: i.e., as a *change* of a mass's velocity in direction or magnitude.) In fact, in his historical treatment of Newton in the *Mechanics*, Mach claimed that Newton had arrived at his definition of "quantity of matter" by actually conjoining masses and their concomitant pressures, not by holding them apart and then (as Mach believed) artificially linking them by introducing a defined term of "force."

According to Mach, Newton should have been surprised by his discovery that bodies possessed the same properties of mass and gravitational attraction, no matter what their nature or inherent chemical differences, as if their differences in composition did not count at the deepest level (Mach, 1960, p. 239). If bodies were different all the way down they should not attract one another equally, particle-by-particle; rather the attraction of the first particle for the second should always exceed or fall short of that of the second particle for the first, if they truly are different in all respects. Only if all bodies were made of the same tiny homogeneous corpuscles should we expect them to exert similar forces on one another. Newton was thus led to regard every unit of attractive force produced by matter as due to the attractions of each indistinguishable corpuscle of matter for one another, and the total attraction of the body was due to the sum total of all its corpuscles' attraction, i.e.,

[24] The law of inertia is included, Mach thought, as the logically equivalent contrapositive of the experimental proposition: where bodies are in no position to exert forces on one another there are no accelerations, hence either rest or motion at a constant velocity (Mach, 1960, p. 172).

the "quantity of matter." According to Mach, then, Newton thought of the quantity of matter as a *pure number* of identical atoms in the body.[25]

Since he is proposing to define mass, Mach mentions not masses but *bodies* ($K_\alpha, K_\beta$) and their accelerations. The mental picture it brings up is of extended lumps of matter floating in space at distances that change according to the accelerations. It would seem, then, that the existence of bodies and accelerations in space were the primary facts, which would give a primary role to spatial measurement. A possible reading of the mass definition is thus one in which Mach began with bodies and their tendency to accelerate one another and then merely defined:

1. what it was for two identical bodies ($K, K'$) to be equal in this tendency, and
2. what it was for a first body ($K$) to be equal in this tendency to a second ($K'$), a second to a third ($K''$) and the first to the third.

On that reading Mach's definition is merely a way of standardizing a capacity of bodies ($K$) to accelerate one another and resist accelerations, a capacity we already knew they had, and for which we simply needed a way of establishing relations of measure. As a reading with limited philosophical interest that is fine.

However, a deeper reading of Mach's definition follows the "Raumvorstellungen" program and has textual support in Mach's writings and notebooks. Recall that the first part of that program was to reduce the spatial relations of extended bodies to the relations between pressures and that Mach had declared his intention to make pressure ($F$) fundamental to mechanics and not derived by a definition ($F = m\mathbf{a}, F = \mathrm{d}\mathbf{p}/\mathrm{d}t$). ($F$ = Force, $M$ = Mass, $\mathbf{a}$ = Acceleration, $\mathbf{p}$ = Momentum.) Mach said in a notebook entry that he intended to measure pressures with pressures:

> There exists something determinative of pressure independent of weight. But it increases and decreases with weight. Proportionality $\mathrm{p = mg}$ $\mathrm{p' = m'g'}$ only by equivalent bodies, or one must already posses the concept of mass, something determinative of pressure independent of material constitution. Measure pressure with pressure. What is pressure. Set it free of subjectivity. Through weight. Mass is: what communicates and receives the m-fold acceleration to an assumed unit body.[26]

Mach hid the distinction between acceleration [*Beschleunigung*] and pressure [*Druck*] in his earlier 1868 article, switching terms midstream. These terms are *not*

[25] Arnold Koslow (Koslow, 1992, pp. 157–163) presents a different case for Newtonian "quantity of matter" based on a controversy between Newton and the editor of the Second Edition of the Principia, R. Cotes. According to Koslow, Newton, urged on by Cotes, eventually did settle on inertia as the measure of mass, anticipating Mach here, and admitted at least the *possibility* of different kinds of matter with equal masses. (Koslow, 1992, pp. 157–163.)

[26] Es existiert ein Druck bestimmendes bei gegebener Beschleunigung unabhängig vom Gewicht. Wächst und nimmt aber mit d. Gewicht ab. Proportionalität $p = mg$ $p' = m'g'$ nur bei gleichen Körpern, oder man muss den Massbegriff schon haben, ein Druckbestimmendes unabhängig von d. materiellen Beschaffenheit. Druck wie d. Druck messen. Was ist Druck. Vom subjectiven frei machen. Durch Gewichte. Masse ist: welcher dem als Einheit angenommene Körper die m fache Beschleunigung ertheilt, welche er selbst erhält. Notebook dated 4 May 1880 (NL 174/2/16). ($M$ = Mass, $p$ = Pressure, Force due to Mass, $g$ = Acceleration of Gravity.)

synonymous: two equal spatial accelerations may represent two very different pressures depending on the masses involved. But this may not be objectionable, since Mach wanted to measure the *combination* of mass and acceleration in pressure, with no independent means to measure the pressures of the accelerating mass unconjoined with the mass itself. But this may have been justified, as all accelerations in mechanics proper are due to the pressures of contact of masses or the pressures of fundamental forces due to mass. Nor is mass measurable except through pressure due to mass, which is why Mach originally complained of a logical circle in mass, acceleration and pressure. That logical circle existed simply because the *definiendum* and *definiens* were part of the same reciprocally defining system of concepts. By accepting *Druck* as a primitive concept, the circularity is avoided, as Mach promised, but the look of mechanics is indeed altered. The problem is now a more general one of how to specialize certain determinations of "a pressure" into more specific concepts corresponding to the mass and to its acceleration.

## 6. Mass as "capacity"

It is an historical irony that just as Newton carried his own corpuscularian views in his pocket, so too Mach held a deeper theory of mass based on an analogy between mechanics and thermodynamics. An explicit mention of this occurred in his article "Eine Bemerkung über den zweiten Hauptsatz der mechanischen Wärmetheorie" (Mach, 1870), where he considered an analogy between the transfer of energies in the impact of masses and the equalization of temperatures between heated bodies. He specified further in his *Principles of the Theory of Heat*:

> The product $\kappa\theta$ of the theory of heat is analogous to the product mv of mechanics. The reciprocal changes of temperature are like the reciprocal changes of velocity, of opposite sign. Neither negative masses nor negative capacities for heat [$\kappa$] have been found. However masses have shown themselves to be independent of velocities, while thermal capacities depend upon temperatures [$\theta$]. Propositions may be constructed for $\kappa\theta$ in one dimension analogous to those for mv in three dimensions (Mach, 1986, p. 180).[27]

Mach generalized the comparison to all energy transfers (but only for reversible processes in equilibrium, not the uncompensated, dynamic, flow of heat between *unequal* temperatures). Speaking of the Carnot-Clausius Law for a reversible energy transfer of heat between temperatures, $(-Q/T + Q'(1/T' - 1/T) = 0)$ where $Q$

[27] Note added by the editor of *Principles of the Theory of Heat* (Dordrecht, Kluwer, 1986), presumably Brian McGuinness: "This statement [i.e. that mass is not dependent on velocity] is now known to be true only of bodies whose velocities do not approximate to velocities of the order of light." We are not told if this note is intended to improve the analogy.

denotes a quantity of heat transformed to work at temperature $T$ and $Q'$ a quantity of heat that drops from a higher temperature $T$ to a lower $T'$, Mach wrote:

> This theorem is not limited to the phenomena of heat, but can be transferred to other natural phenomena, if, instead of the quantity of heat, we put the potential of whatever is active in the phenomenon, and, instead of the absolute temperature, the potential function…If, for example, we wish to apply the theorem to the impact of inert masses, obviously the vis viva of these masses is to be conceived as the potential and their velocity as the potential function. Masses of equal velocity can communicate no vis viva to one another—they are at the same potential level (Mach, 1871a, pp. 85–86).

According to this manner of viewing physics, every energy is expressible as a product of two factors: a capacity factor which indicates how great an agent is acting and an intensity factor which indicates how intensely that agent is acting (Hiebert, 1962). Mach indicated in his 1870 article that he had considered the analogy for all energy forms, which in his time would have meant mechanical phenomena, thermodynamics and electromagnetism, which were also the main contenders for a general meta-scientific world view in the late 19th century. Mach was often associated with the energetics movement, as represented by Georg Helm and Wilhelm Ostwald (see Deltete, 1983). And although there are similarities, Mach differed from them on two essential points. Mach did not believe that energy, space and time were the ultimate dimensions of physics—space and time being derived according to him. And unlike Ostwald, Mach did not believe in the concept of energy as a kind of enduring, universal substance underlying all change, as Hiebert writes in his article on Mach for the Dictionary of Scientific Biography:

> For Mach the principle [of the Conservation of Energy] took the position of a maxim or convention for organizing a large class of natural phenomena and was rooted in an anthropomorphic sanction related to a biologically determined economy of effort conducive to survival. ‘‘Energy’’ was for Mach no more than a plausible and powerful concept like force, space or temperature. Thus it is wrong to include Mach among energeticists such as Wilhelm Ostwald and Georg Helm—as is often done.[28]

Helm, too, believed the energy law to be merely a technique for relating the appearances, not a new view of substance. But unlike Helm, Mach modified his belief in the conservation of energy in the light of the second law of thermodynamics, arguing that if energies converted to heat could not be cyclically re-obtained in work or its equivalents, it was improper to speak of energy conservation in such cases (Mach, 1986, pp. 318–319).

Mach's treatment of natural phenomena stays close to classical potential theory and seems remote from the terminology of classical thermodyamics, at least as it was

[28] Erwin N. Hiebert ‘‘Ernst Mach’’ *Dictionary of Scientific Biography* (New York: Scribners, 1973) p. 598; cf. Hiebert *The Conception of Thermodynamics in the Thought of Mach and Planck* (Freiburg: Ernst Mach Institut, 1968).

interpreted by the energetics movement. For example, the volume energy which seems to have been central to Helm (see Deltete, 1983) was not so for Mach. Working from the example of mass and distance in mechanics, Mach saw each natural energy as having a source, which he called simply the potential, and which could be a charge, a lodestone, a mass or a heat source. Mach also saw that in each case a potential function could be found, which described the intensity with which the source could potentially affect another source. Hence Mach tended to concentrate not on the mass per se, but always on the mass in so far as it possessed a certain potential in relation to other masses because of its distance or its velocity. Mach treated all of the natural potentials this way, focusing not of the constancy of the quantity of mass, heat or electricity but always on a quantity *at a certain potential level* to produce energy changes either in that same form or in transformations to other forms (Mach, 1911, pp. 43–46). Mach called the temperature, the velocity, the distance between masses the "potential function" because the energies exchanged were a function of the difference between their "potential levels." Quantities of heat can only do work as a function of passing between warmer and colder (absolute) temperatures. A mass only conveys energy by impact with a velocity difference; the attraction of a mass for another is greater as the separation decreases.

In Mach's treatment of the kinetic energy, mass became a capacity factor and velocity (or the squared velocity[29]) the intensity factor. In his treatment of gravitational potential, mass is again the capacity factor and the spatial separations of the masses are the intensity factor in their interaction. Mach's description abstracted completely from spatial properties of the different energies. A position, a temperature, a squared velocity, the difference in the spatial properties of these quantities made no difference to Mach. Even the fact that velocity was a vector and temperature a scalar did not obviate the analogy.

In the *Mechanics* Mach says explicitly that he arrived at the mass definition by investigating the "interdependence of phenomena," and by comparing mass in mechanics to quantity of electricity and quantity of heat (Mach, 1960, p. 267). Thermodynamic analogies occur in an opening section of the 1868 mass paper where Mach was trying to show that the principle of inertia was not a priori, for it could always be imagined that masses did not induce accelerations in one another, but rather velocities, as in equalizations of temperature (that this is the appropriate property to which to compare accelerations presupposes the validity of the analogy which the comparison itself belies):

> If two masses opposite one another gave one another not accelerations, but rather perhaps velocities dependent on the distances, then there would be no law of inertia. Only experience teaches whether one or the other takes place. If we had only sensations of warmth, then there would only be equalizing velocities, which would be zero when the temperature differences were themselves = 0 (Mach, 1868, p. 356).

[29] See Mach (1911), p. 94: "I soon recognized that the potential level is a scalar $\mathbf{v}^2/2$, and cannot be a vector $\mathbf{v}$ or $\mathbf{v}/2$."

Equalizations of temperature between a colder and a warmer body in contact occur at a rate proportional to the temperature differences themselves. Gravitationally attracting masses equalize their potential differences (their differences of position) by closing the separation between them at an accelerated rate proportional to the square of the time. Differences of position between two mass-potentials induce changes of changes of those positions, whereas heat tends to equalize temperatures by flowing at a constant velocity, proportional to the temperature differences.

For Mach distances covered by bodies in uniform motion are analogous to "constant flows" of heat from higher to lower temperatures, a case Mach called "dynamic equilibrium." Mach summarized the view later in the *Mechanics*. Just as the position of a body moving at a constant velocity with reference to surrounding bodies is the mean of its earlier and later positions (its potential levels relative to other attracting masses), so too a constant flow of heat passes between temperatures that are the mean of adjacent temperatures.[30]

> That accelerations play a prominent role in the relations of the masses, must be accepted as a fact of experience; which does not however exclude attempts to elucidate this fact by a comparison of it with other facts, involving the discovery of new points of view. In all processes of nature the differences of certain quantities u play a determinative role. Differences of temperature of potential function and so forth, induce the natural processes which consist in the equalization of those differences. The familiar expressions $\partial^2 u/\partial x^2$, $\partial^2 u/\partial y^2$, $\partial^2 u/\partial z^2$, which are determinative of the character of the equalization may be regarded as the measure of the departure of the condition of any point from the mean of the conditions of its environment—to which mean the point tends. The acceleration of masses may be analogously conceived (Mach, 1960, pp. 288–289).

In Mach's conception of the physical processes, the system of masses standing apart at distances was replaced by a set of capacities for attraction standing at their various potential levels with respect to one another, and moving from one level to another to equalize those differences. Thus, when Mach speaks of accelerations in the above passage of the *Mechanics* it is only in the sense of an equalization of level that manifests itself as a change of a change of a difference of potential level in unit time. For him, this thermodynamic conception of acceleration had completely overtaken the Galilean concept of changes of velocity. In his 1896 *Theory of Heat*, Mach explicitly developed a definition for isolating a body's capacity for heat by an experimental procedure analogous to his definition of mass (Mach, 1986, p. 178).[31]

[30] Keeping in mind Mach's functional presentation of physical dependencies, "adjacent" has the sense of causal proximity. $U$ is what Mach calls the "potential function" $U(x, y, z)$ in Cartesian coordinates.

[31] Mach would eventually be criticized by Planck and Boltzmann (mostly Planck) on this analogy between temperatures and quantities of heat and heights of level and masses. Both Planck and Boltzmann believed that the flow of heat from warmer to colder bodies was a completely different physical phenomenon. In the case of Planck (at the end of the 19th century and before his fundamental work on black body radiation) flows of heat were more analogous to other irreversible processes in nature, many of which did not involve differences of potential level at all, for example the irreversible mixing of two gases or fluids. For Planck, the reversible processes exhibited by transformations of the other energies were in an

## 7. Mass and inertia

Mach's revisions to the Newtonian principle of inertia were also evident in the 1868 mass paper. For example, he made good on his claim to make the principle an empirical theorem:

> Bodies placed opposite one another communicate to each other accelerations in opposite senses in the direction of the line of junction. The law of inertia is included in this (Mach, 1868, p. 358).

Why is the law of inertia "included" in the law of universal gravitation? Mach's implicit assumption is that all bodies would be continually accelerating one another if they could, thus it is only in special circumstances, when all of the forces on a body cancel in magnitude and direction, that the body is at rest or moves at a constant velocity. These are specialized conditions of equilibrium that presuppose dynamical relations between a body and *all* surrounding masses that might tend to accelerate it.[32] As Mach wrote as early as his 1872 end-notes to the *Conservation of Energy* (Mach, 1911, pp. 78–80), this can occur in local environments, such as the solar system, and in other circumstances when a mass stands at a great enough distance from the other masses of the universe. However, Mach adds, should the universe "swarm in confusion," the law of inertia would be revealed as the special equilibrium of forces that it is: a calm eye in the storm of competing pulls. He also believed that invoking the masses of the universe as the system of reference solved the paradox that a velocity can preserve its 3-component direction even when it is not evident what the directions are reckoned towards. The direction of a velocity is three-fold since motion at a constant velocity is a special case of a three-fold dynamical relationship, of acceleration.[33]

It is well known that Mach resisted all speculation about a mechanism, or even a need for one, that could convey gravitational influences instantaneously, or

(*footnote continued*)
entirely different class and any further similarities among energy forms were purely incidental. In Boltzmann's case it was more a question of the statistical nature of the second law, over against the iron necessity of the first. Eventually Boltzmann converted Planck over to his view (see Erwin Hiebert "The Conception of Thermodynamics in the Scientific Thought of Mach and Planck" Freiburg: Ernst Mach Institut, 1968, On Planck's attack on the analogy between falling weights and heated bodies see Robert Deltete (1983), pp. 563–577).

[32] A contemporary improvement on Mach's idea to make dynamics precede statics is Julian Barbour's delightful suggestion that Minkowskian space–time be viewed as a special case of the dynamical general theory of relativity. "What we need is a change of perspective—away from the view that flat Minkowskian space is the "natural ground state of the universe" (or, alternatively, a fixed external framework) and to the recognition that it is a highly atypical solution of a sophisticated dynamical theory." In "General Relativity as a Perfectly Machian Theory" *Mach's Principle: From Newton's Bucket to Quantum Gravity: Einstein Studies Vol. 6* (Boston: Birkhäuser), pp. 226–227.

[33] In cases where a body acquires and maintains a velocity, and hence vis viva that can be communicated by impact, Mach spoke of a body's "velocity potential" and interpreted the squared velocity as its potential level for vis viva, where once again the mass is the capacity factor. The transfer of gravitational potential energy into the kinetic energy of moving bodies was viewed by Mach as a transfer from one form of energy to another, just as if gravitational potential energy had, instead, led a body to become electrified.

at a velocity such as that of light, from the fixed stars to a body in a local neighborhood like the solar system (Mach, 1960, p. 296). John Norton sums up his review of Mach's own writings on the subject of "Mach's principle" by saying:

> …the only unequivocal proposal is that we eliminate the odious notion of space by redescribing the relevant experiment and law [of inertia] in a way that does not use the term 'space.' If there is a suggestion of a new physical mechanism that would reach from the distant stars to cause the inertial forces in Newton's bucket, then the proposal is made vaguely and we are left to wonder whether Mach endorses it or condemns it as unscientific (Norton, 1995, p. 45).

The assessment is correct of course, but Norton overlooks Mach's thermodynamic conception of the acceleration of masses—which is mentioned in the conclusion to the very sections of the *Mechanik* he considers (Mach, 1960, pp. 288–289 also quoted above). The thermodynamic conception was Mach's *only* endorsed view of the natural processes and remained so throughout his life (Mach, 1976, p. 357). Mach considered the facts of physics to be essentially described by the natural potentials, their potential functions and special laws governing the energy transformation; nothing else. In particular, an appreciation of Mach's non-Galilean theory of acceleration (as equalizations of potential level) appears to me to be crucial to any interpretation of Mach's principle according to Mach. The hypothesis of spatially propagating actions (Wirkungen) from one body to another, later invoked by Einstein[34] to advance or explain Mach's revision of Newtonian inertia was probably seen by Mach as a backward step into picture thinking and not an advance toward the future physics of pure elements and functions that he favored. Even if the agency of inertial forces is due to the stretching of a colloidal medium surrounding the bodies like an aether—a suggestion Mach feinted towards in the Mechanics and in a letter to Josef Petzoldt[35]—still overall Mach clearly considered the explanatory style of seeking mechanisms for basic laws a mistake. Mach's low opinion of aether theories, for example, was given in his lecture Allgemeine Fragen der Naturwissenschaft (Sommer, 1897):

> Magnetic currents, gravitational currents. Only analogies. Pictures. Exactly like caloric. Electricity as a fluid in which no one seriously believes.[36]

[34] I am referring to the idea that: "the whole inertia of any material point is an effect of the presence of all other masses, depending on a kind of interaction with them" which comes from Einstein's 1912 paper "Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?" *Vierteljahresschrift für gerichtliche Medizin und öffentliches Sanitätswesen* **44** (37–40). It would be interesting to know what Einstein thought of Mach's analogies between mechanics, electricity and heat.

[35] Mach to Petzoldt September 18, 1904 in Blackmore and Hentschel eds. 1985.

[36] Allgemeine Fragen der Naturwissenschaft Sommer 1897 (NL 174/1/019): Magnetische Ströme, Gravitationsströme. Nur Analogien. Bilder. Gerade so gut wie d. Wärmestoff. Electricität als Flüssigkeit an die niemand in Ernst glaubt.

## 8. Mass: a standardized capacity factor

Taking all of the above results in hand, a likely reconstruction of Mach's reasoning behind the mass-definition runs as follows. There is no such thing as a body that does not accelerate according to universal gravitation; even inertial motions are just special cases of accelerations. Thus all bodies will press one another according to their capacity and intensity factors for inducing pressure. We can feel the pressures of contact resident in the body by trying to accelerate it, or collide with it, and we can feel the pressures due to fundamental forces by bringing sufficiently large bodies together.

The constant conjunction of force with matter thus leads us to believe that there is a capacity for acceleration (or resisting it) resident in matter independent of its chemical composition or other properties we discover, such as magnetism and electricity. Mass was but *one* of a number of capacities for energy possessed by bodies (Mach, 1960, p. 240). Thus, we need to *isolate* this particular capacity from other capacities of a body due to its peculiar chemical composition.

Once the capacity for acceleration has been isolated we discover the fact that if two bodies are equal in their capacity to accelerate one another and one of them is equal to a third, the other is also equal to this third. The separation of this capacity from the specific composition of bodies is the first thing Mach's definition aimed to prove. The mass concept thus represents a standardization and metricization of capacities that we already know they have. Mach's definition isolates the capacity factor of mass and gives it a unique meaning, even though it does not appear alone in experience, independent of its intensity factor of distance. This is why he insisted that mass be defined first for two identical bodies $m,m$, and at a fixed distance from one another: that holds steady both the capacity and intensity factors. Then we can consider extending the concept beyond identical bodies to those of heterogeneous chemical compositions.

But for this extension we need an argument. Thus, Mach says that if bodies did not press one another equally, either by attraction or by contact, then it would be possible to construct a perpetuum mobile. Take the pressures produced by contact. We set three bodies on a frictionless ring and start one in motion (Mach, 1868, p. 358). If the collision produces more vis viva in the second body than was in the first, and more in the third than was in the second, then when the machine goes through one complete cycle and the vis viva is communicated back to the first body, there will be more kinetic energy in the ring than there was to start with, a perpetuum mobile. Mach's proof mimicked exactly (and is probably patterned on) Sadi Carnot's argument that the quantity of heat exchanged between two bodies depends only on their temperatures and not their chemical compositions (Mach, 1986, pp. 203–204).

Mach assumed that two absolutely *equal* bodies at a distance from one another (a fixed difference in potential level), will produce equal and opposite pressures, which could have been measured by means of motionless test-pressures:

$$F_{\alpha\beta} = -F_{\beta\alpha}.$$

Rather than assuming $F = ma$, Mach then assumed some separation between the experimentally fixed capacity and intensity factors so that he could analyze pressure into masses and accelerations:

$$m_\alpha \varphi_{\alpha\beta} = -m_\beta \varphi_{\beta\alpha}$$

or

$$m_\alpha / m_\beta = -\delta_{\beta\alpha} / \delta_{\alpha\beta}.$$

In these physical speculations, treating distance, acceleration and velocity in a very different way than the concepts are used in Galilean mechanics, Mach took certain concrete steps towards the realization of his 1866 "Raumvorstellungen" program. The "states of matter" he referred to in the "Raumvorstellungen" paper are pressures $(F, F', F'' \ldots)$ due to contact and fundamental forces. These are the basic elements in their reciprocal functional dependence on one another.

The true problem of the mass definition now appears considerably simplified via the philosophical program. We must get from the experimentally observed pressures to two specifications of pressure: those attributable to constant potential sources inducing (resisting) acceleration and those due to constant potential function. Because a mass is always a mass at a potential level for Mach, these two factors are fundamentally entangled and cannot occur separately. However it is possible to separate them in thought by applying two different standards of comparison: a variety of mass-capacities $(m, m', m'' \ldots)$ at the same potential level $(r)$; the same mass-capacity $(m)$ taken at a variety of constant potential levels $(r, r', r'')$. A further principle specifies that the equalizations of level $(r, r', r'')$ are manifested as accelerations and not velocities as in the case of heat. For a definition of mass, a technique is required for fixing both factors independently, which is provided by the methods of comparing test bodies and distances together in experimental physics.

## 9. Mach's later remarks on the mass definition

Mach reflected on his mass definition in letters to his friends. Here he seemed to be trying to convey the notion that the mass definition was similar to a concept of measurement or of space. I interpret this to mean that his definition standardizes the capacity factor of mass through the interactions and exchanges of energy between bodies, the same way that a metric definition for space standardizes a certain length through motions from place to place.

In a letter to Josef Popper–Lynkeus of December 30, 1883 (right after the first edition of the *Mechanik*) Mach wrote:

> Concerning mass I am convinced that I have hit the mark in not making this a fundamental concept. It is not a concept of experience like length, but rather a concept of space (Blackmore and Hentschel, 1985, Letter #3).

Mach also corresponded extensively with Friedrich Adler about the latter's own mass-concept and how it differed from Mach's:

> For me the concept of mass is a dynamical concept of measure [*Massbegriff*], which as such assumes a relation to a comparison body and nothing else. When bodies dissolve, evaporate and so forth, this can cause difficulties in the application of the concept but does not affect the concept itself. Everywhere that the concept of mass or force presents itself in physics, only the mechanical side of the process is meant; only ponderable masses come into consideration, and reciprocal action against another ponderable body must take place. I have already said for 30 years that the validity of the same measure of the masses [*Massenmasses*] for arbitrary physical circumstances is an hypothesis that has been valid until now…When we speak of the conservation of "mass" or the conservation of "matter" that can only mean one thing: in all changes of a closed system the sum of the mass-values of the bodies, in relation to one determinate comparison-body remains constant. In "matter" I see nothing but the proportionality of all physical capacities [*Capacitäten*] to one another (Blackmore and Hentschel, 1985, Letter #22).

What that last sentence seems to mean is that bodies are also thermal, electric and gravitational sources, and that matter is simply the proportionality of these through transformations of the energy from one form to another. Adler wrote back on January 6, 1904 with an admirable précis of Mach's own mass concept, declaring that the system of masses is only thought of spatially as a result of considerations of economy:

> Masses are, as constant relational values, a system of numerical magnitudes and nothing further. We take a new step when we affix [*zuordnen*] this system to a system of bodies. In every movement of one body we have to ascribe the mass value a new place. The states (temperature, color) that form the bodies may change but we will affix to the body the same mass-value, as long as the accelerations that occur in them point to it. Bodies change; all states occur discontinuously [*unstetig*] in space (changes of energy occur). The masses however move constantly in space, or, when we cannot establish this, the assumption of a constant motion appears to us the most convenient. The system of masses that is thought of spatially only out of economical considerations proves itself the most appropriate, in order to relate it to all other appearances. Before the conception of masses as a spatial system, whose arrangement constantly changes, there is no reason to designate this relational value as the capacity (Blackmore & Hentschel, 1985, Letter #23).

## 10. Mach's late view of matter: a chemical manifold

The "Raumvorstellungen" program concluded with Mach's late theory of matter and space, developed in writings from 1896 to 1910. Here he advanced the rest of the way toward the elimination of extended properties by reducing all ponderable bodies to states within a "chemical manifold." As he says already in a notebook fragment of 1886 "space is a three-fold chemical change."[37] By "chemistry," Mach simply meant a fully general science that considers general transformations of energy without regard to what physical department they belong, or, as he put it, a change of chemical potential level encompasses a manifold of other physical potential levels (Mach, 1986, pp. 329–331). A "chemical manifold" in the late writings was for Mach a Riemannian manifold in which all of the physical energies and their transformations could be simultaneously manifested, a conception he discussed in his *Knowledge and Error* and worked on continually in lectures and notebooks. He even imagined a volume element of this manifold to correspond physically to the notion of a capacity factor across different energy manifestations:

> The same role for the conservation of space in geometry as the continuity and conservation of masses in physics.[38]
>
> Homogeneous volumes = Capacity.[39]

In these ideas of really deriving space from the physical potentials rather than representing potential functions as themselves functions of time and spatial variables, Mach seems to have returned to the Herbartianism of his youth. The spatial factors (bodies, distances) seem to arise as specialized orders of constant potential, potential function, as Herbart made space and time arise from his intensive qualities and *Wesen*. The connection is very explicit in Mach's lectures,[40] and in *Knowledge and Error*:

> Our intuitions of space and time form the most important foundations of our sensory view of the world and as such cannot be eliminated. However this does not prevent us from trying to reduce the manifold of qualities of place-sensations to a physiological-chemical manifold. We might think of a system of mixtures in all proportions of a number of chemical qualities (processes). If such an attempt were one day to succeed, it would lead also to the question whether we might not give a physical sense to the speculations that Herbart, following Leibniz, conducted as regards the construction of intelligible space, so that we might reduce physical space to concepts of quality and magnitude. There is of course much to be objected to in Herbart's metaphysics. His tracking down of contradictions that are in part artificially contrived and his eleatic tendencies are none too attractive, but he will hardly have produced nothing but errors (Mach, 1976, p. 349).

[37] "Dreierlei chemische Änderungen sind der Raum" (Notebook dated 2 January 1886 (NL 174/2/23)).

[38] Die Raumerhaltung dieselbe Rolle in der Geometrie, wie die Continuitat u. Massenerhaltung in d. Physik (Notebook dated 9 November 1900 (NL 174/2/43)).

[39] Homogene Volumen = Capacität (Notebook dated 14 July, 1907 (NL 174/2/51) underlined in original).

[40] These remarks accompany his exposition of "chemical manifoldness": Herbart intelligibler Raum. Riemann. Helmholtz. Allgemeine Fragen der Naturwissenschaft Sommer 1897 (NL 174/1/019).

Mach's chemical manifold thus brought together his conceptions of physiological and physical space more closely than at any other time in his development.[41] Throughout most of his career, Mach was a *nativist* about physiological space, believing that it was due to an innate organization of the sense organs, such as the retinal surfaces, the skin and especially the motor apparatus. In his physical science from the period of ca. 1866–1890, Mach was an *empiricist*, believing, as we have seen, that spatial properties rested on a set of facts about measurement, which in turn reduced to aspatial *functional* dependencies of elements on one another. In his notion of physiological-physical space, however, Mach began to see that certain physiological properties such as the unidirectional flow of memories and thought or the left, right asymmetry of the body and even three-dimensionality could be represented by chemical properties shared by physical space.[42] He was especially impressed with the fact that the "right hand rule" could be used to determine the direction of an electric current given the direction of the magnetic field (the curled fingers of the right hand point in the direction of the field and the thumb points in the direction of the current). Physical facts of the time did not serve to determine the direction of the current, but a physiological asymmetry allows the determination to be made, provided we can tell our right hands from our left (this task is not self-evidently easy for me by the way unless I make two "L" shapes with my hands and remember that the one that spells L is "left").

In a quite advanced sense, Mach allowed that *certain properties* of physiological space (dimensionality, anisotropy) had correlates in physical space:

> Physical space also has three essentially different directions, which are most clearly manifested in a triclinal medium, in the behavior of an electro-magnetic element. The same physical properties appear also in our own body, which is the reason why our bodies can be used as reagents in physical problems. If we had an exact physiological knowledge of an element of our bodies we should thereby have laid, in all essentials, the foundation of our understanding of the physical universe (Mach, 1959, p. 339).

In particular, Mach seemed to think that the directions of space represented real physical properties, as if the natural potential themselves were the fundamental spatial directions determining the multiple extension of whatever filled space. (See Banks, 2003a, for details of this late view as well as Mach's "Eine Betrachtung" see note 42.)

Even given this convergence of physical and physiological properties in his late views, I think the general remark is justified that, beginning with his earliest work, Mach intended to lead the way towards a future physics mature enough to analyze space into simpler properties, be they the elements and functions of the empiricist stage or the chemical properties and directions of the late stage.

[41] See the final chapter of Banks (2003a) for a relatively plausible reconstruction of space "by chemical paths" in Mach's late writings.

[42] "Eine Betrachtung über Raum und Zeit" (in Mach 1987, pp. 497–499).

Mach's ideas are still rich and philosophically interesting today, if only for investigating how to reduce the concept of extension to simpler intensive factors, but they proved impossible to motivate as physics, I believe, due to the inherent impracticality of reformulating physical concepts in an aspatial format without a really compelling physical problem to demand it. The problem of the spectral lines did not end up being decisive in the way Mach thought. And even Mach's principle of inertia and definition of mass did not lead to an attempt to reduce spatial properties in physics, as he had hoped, but rather seems to have led Einstein (at least in 1912) and others to formulate Mach's aspatial ideas in their own spatial terms.[43]

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[43] Mach said in the very controversial preface to his *Optik* dated July 1913 that it was "above all his correspondence" that forced him to distance himself from the theory of relativity. Sure enough, shortly before the date of the *Optik* preface, Mach received two letters, one from Joseph Petzoldt on June 15, 1913, the other from Einstein June 25, 1913, both of which attribute to Mach the idea that the distant masses affect a local body by means of a dependency (Petzoldt) and a reciprocal action or Wechselwirkung (Einstein) (Blackmore and Hentschel, 1985, pp. 120–121).

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