

“Newton’s Scholium on Time, Space, Place and Motion”

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Introduction

There is no question that Newton’s famous bucket experiment radically altered the landscape in which philosophical debates over the nature of space were carried out. What exactly Newton intended to show with his experiment is, however, still a point of considerable controversy. In this essay, I will argue that Newton’s intentions were more modest, but also more sensible, than has been commonly supposed.

The essay itself falls into three parts. The first briefly sketches the original experiment, as well as, its traditional interpretation in light of Ernst Mach’s discussion in *The Science of Mechanics*. The second, highlights the Cartesian background to the bucket experiment and argues that, once it is properly viewed as an attack on Descartes’s distinction between common and philosophical motion, the mistake attributed to Newton by Mach simply falls away. The third section considers the possibility that Mach’s objection might nonetheless reassert itself in connection with Newton’s almost equally famous two globes thought experiment.

1. The Bucket Experiment and Mach’s Interpretation

In the third to last paragraph of his scholium on time, space, place and motion, Newton suggests that, at least in some circumstances, absolute motions may be distinguished from

relative motions by their observable effects. In illustrating his claim, he introduces his famous bucket experiment as follows:

If a bucket is hanging from a very long cord and is continually turned around until the cord becomes twisted tight, and if the bucket is thereupon filled with water and is at rest along with the water and then, by some sudden force, is made to turn around in the opposite direction and, as the cord unwinds, perseveres for a while in this motion; then the surface of the water will at first be level, just as it was before the vessel began to move. But after the vessel, by the force gradually impressed upon the water, has caused the water also to begin revolving perceptibly, the water will gradually recede from the middle and rise up the sides of the vessel, assuming a concave shape (as experience has shown me), and, with an ever faster motion, will rise further and further until, when it completes its revolutions in the same times as the vessel, it is relatively at rest in the vessel (Newton [1726] 1999, 412-413).

Newton draws attention to four stages of the experiment. The first occurs at the start when the bucket has been filled with water, but the cord has not yet been permitted to unwind. At this initial stage there is no relative motion between the water and the bucket, and the surface of the water is flat. The second stage occurs when the cord first begins to unwind. There is now a relative motion between the water and the bucket even though the surface of the water is still flat. At the third stage there is likewise a relative motion between the water and the bucket, but the water has only just begun creeping up the sides deforming the surface. The fourth, and last stage considered by Newton, occurs when the bucket and the water have the same angular velocity. At this point, the surface of the

water remains concave even though there is no longer any relative motion between the water and the bucket.

At the end of the bucket experiment, Newton appears concerned only to draw a negative conclusion, namely, “that endeavor [of the water to recede from the center] does not depend on the change of position of the water with respect to surrounding bodies, and thus true circular motion cannot be determined by means of such changes of position” (Newton [1726] 1999, 413). Newton’s commentators, however, have traditionally read the bucket experiment (as well as the two globes experiment occurring two paragraphs later) as contributing to a larger argument for the existence of absolute space. Thus according to Ernst Mach’s influential reading, Newton is first supposed to have inferred that the forces manifested in the water’s surface do not correlate with the relative motion between the water and the bucket, and second that they must therefore be correlated with the water’s motion relative to absolute space. It is the *non sequitur* of the second step that Mach famously criticizes in his *Science of Mechanics*:

Newton’s experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick. The one experiment only lies before us, and our business is, to bring it into accord with other facts known to us, and not with the arbitrary fictions of our imagination (Mach, [1908] 1989, 284).

Mach's reading of the bucket experiment became accepted doctrine with later day philosophers of science who shared his positivist intuitions. Thus in his influential *Space and Time*, Hans Reichenbach writes:

Newton concludes that the centrifugal force cannot be explained by a relative motion, since a relative motion exists between the pail and the water at the beginning as well as at the end . . . Mach replies that Newton overlooked the fact that the surrounding masses of the earth and fixed stars have to be taken into consideration. The water rotates not only relative to the pail but also relative to these large masses, which may be considered as a cause of centrifugal force (Reichenbach 1957, 213-14).

Ernest Nagel sounds the same note in his widely read *The Structure of Science*:

Newton's argument was severely criticized by Ernst Mach, who showed that it involved a serious *non sequitur*. Newton noted quite correctly that the variations in the shape of the surface of the water are not connected with the rotation of the water relative to the sides of the bucket. But he concluded that the deformations of the surface must therefore be attributed to a rotation relative to *absolute space*. However, this conclusion does not follow from the experimental data and Newton's other assumptions, for there are in fact two alternative ways of interpreting those data: the change in the shape of the water's surface is a consequence either of a rotation relative to absolute space or of a rotation relative to *some system of bodies different from the bucket* (Nagel 1961, 209).

The Machian criticism of Newton, of course, bore its own fruit in connection with Einstein's development of the theory of general relativity. Its credentials as fair criticism

of Newton, however, have more recently come under fire (Laymon 1978, Earman 1989, 61-66, Rynasiewicz 1995a and 1995b). In the next section we will consider how the Cartesian background to the bucket experiment clears Newton of the charge leveled by his machian interpreters.

2. The Bucket Experiment in Context

In his *Principles of Philosophy*, Descartes distinguishes between common and philosophical motion. The former he explains:

. . . is nothing other than the action by which some body travels from one [relative] place to another. And, therefore . . . the same thing can be said to simultaneously change, and not change, its place; so it can also be said to move and not to move. Thus a man, seated in a ship which is sailing out of port, thinks that he is moving if he turns his attention to the shores, which he considers to be at rest. But he does not think so if he turns his attention to the parts of the ship, in relation to which he constantly maintains the same situation (Descartes [1644] 1984, Pr II 24).

Our workaday concept of motion is thus, according to Descartes, a relative one. Whether or not a body is said to move by the vulgar depends upon the selection of an arbitrary set of coordinating bodies. It is therefore perfectly possible for a body to move (relative to one set of bodies) and nonetheless to be at rest (relative to another set of bodies).

This common conception of motion is contrasted in *Principles* II, 25 with a true or philosophical conception of motion. There, Descartes explains:

If, however, we consider what should be understood by movement, according to the truth of the matter rather than in accordance with common usage (in order to attribute a determinate nature to it): we can say that it is the transference of one part of matter or of one body, from the vicinity of those bodies immediately contiguous to it and considered as at rest, into the vicinity of [some] others (Descartes [1644] 1984, Pr II 25).

Although somewhat obscure in the details, Descartes's definition suggests that true motion must be understood as a body's motion relative to its contiguous and immediately surrounding neighbors. Unlike common motion, true motion is thus uniquely determined – on Descartes's definition a body is either truly moving or it is not.

The five paragraphs of Newton's scholium, bracketed on the one end by the ninth paragraph which starts "Moreover, absolute, and relative rest and motion are distinguished from each other by their properties, causes, and effects," and on the other end by the paragraph containing the bucket experiment, are most cogently read as a sustained critique of the Cartesian distinction between common and true motion. The first three of these five paragraphs all suggest in one way or another that the Cartesian distinction must be rejected because it fails to guarantee an intuitively essential property of true motion. So, for example, in the tenth paragraph of the scholium, Newton writes:

Therefore, when bodies containing others move, whatever is relatively at rest within them also moves. And thus true and absolute motion cannot be determined by means of change of position from the vicinity of bodies that are regarded as

being at rest. For the exterior bodies ought to be regarded not only as being at rest but also as being truly at rest. . . . For containing bodies are to those inside them as the outer part of the whole to the inner part or as the shell to the kernel. And when the shell moves, the kernel also, without being changed in the position from the vicinity of the shell, moves as a part of the whole (Newton [1726] 1999, 411).

Newton's objection is that any reasonable definition of true motion must respect the fact if a body A is at rest relative to a body B, then if A truly moves, B must truly move as well. To deny this would be to open the door to such absurd consequences as the possibility that an envelope might truly move in going from California to New York without the letter inside ever truly moving at all. As Newton points out, however, Descartes's definition seems to leave just this sort of possibility open. For it would seem, according to Descartes's definition, that if body B surrounds body A, A may remain truly at rest – because it does not move relative to its surrounding body - even as B truly moves while carrying A inside of it.

In the twelfth paragraph of the Scholium, Newton raises a second kind of objection to Descartes's distinction between common and true motion. He suggests that in addition to respecting our intuitions concerning the properties of true motions, an adequate definition of absolute motion should recognize that the generation and alteration of true motions must be correlated with the forces taken to act on bodies. He writes:

The causes which distinguish true motions from relative motions are the forces impressed upon bodies to generate motion. True motion is neither generated nor changed except by forces impressed upon the moving body itself, but relative motion can be generated and changed without the impression of forces upon this

body. For the impression of forces solely on other bodies with which a given body has a relation is enough, when the other bodies yield, to produce a change in that relation which constitutes the relative rest or motion of this body. Again, true motion is always changed by forces impressed upon a moving body, but relative motion is not necessarily changed by such forces. For if the same forces are impressed upon a moving body and also upon other bodies with which it has a relation, in such a way that the relative position is maintained, the relation that constitutes the relative motion will also be maintained. Therefore, every relative motion can be changed while the true motion is preserved, and can be preserved while the true one is changed, and thus true motion certainly does not consist in relations of this sort (Newton [1726] 1999, 412).

The quoted passage suggests a second intuitive constraint on any definition of true motion, namely, that true motions can only be generated or altered by the application of force (cf. Newton [1668?] 1962, 91-92, 96, 103). Such a constraint would have, of course, seemed entirely wrongheaded to generations of natural philosophers prior to Galileo, and even to many after. Most proponents of the medieval impetus theory, for example, implicitly rejected it, assuming that a constant true motion requires a constant force.

Descartes himself, however, argues for a principle of inertia that seems to embrace just the constraint in question. In articulating this parameter, Newton's intention therefore seems to be to point out that it is violated by Descartes's own definition of true motion. For if, again, we suppose that body B surrounds body A, then we (or God) might generate a true motion in A merely by applying a force to B. Such a reading of Newton's

intentions is supported by a related passage from his unpublished essay *De gravitatione*, where he writes:

It follows from the Cartesian doctrine that motion can be generated where no force is impressed. If, for the sake of argument, God were to make it happen that the rotation of our vortex were suddenly to stop, without impressing on the Earth a force which would stop it at the same time, Descartes would say that, because of its translation from the vicinity of the contiguous fluid, the Earth would not move in the philosophical sense, just as before he said it be at rest in the same philosophical sense (Newton [1668?] 1962, 95).

According to Newton, Descartes's definition thus fails to distinguish true from merely relative motion, not only because it ignores an essential property of true motion, but also because it does not succeed in capturing the intuitive correlation between true motion and its causes.

The bucket experiment, found in the twelfth paragraph, continues the line of argument initiated in the ninth paragraph by suggesting that the Cartesian distinction between common and true motion fails to correspond intelligibly with the observable effects of circular motion. In the background here are Descartes's second law of nature, as well as, his theory of cosmological vortices. Descartes's second law states, "that all movement is, of itself, along straight lines; and consequently, bodies which are moving in a circle always tend to move away from the center of the circle they are describing" (Descartes [1644] 1984, Pr II 39). The theory of cosmological vortices in turn serves to explain a wide variety of astronomical phenomena within the Cartesian system – including the sun's illumination and the planets' orbits – by appeal to the centerfugal

endeavor described in the second law and supposedly induced by the circular motion of celestial matter.

With this background, the bucket experiment can be seen as a strikingly precise attack upon the Cartesian distinction between true and common motion. Assuming that the effect described by Descartes as a centerfugal endeavor correlates with the deformation of the water's surface, Newton is able to argue that the Cartesian definition of true motion makes a hash of the observable effects of circular motion. For according to that definition, the water is truly at rest during the first and fourth stages, and truly moving during the second and third stages. But in that case there is no sensible correlation between the true motion of the water and the deformation of its surface, for, as we have already noted, the water's surface takes on a paraboloid shape only in the third and fourth stages.

Here, as elsewhere, it is possible that Descartes might be able to mount some sort of response to Newton's criticism. With regards to natural philosophy, after all, Descartes was nothing if not ingenious. Nonetheless, what should be clear from the preceding is that the Machian objection to the bucket experiment misses its mark as a criticism of Newton. For as an attack on Descartes's definition of true motion, the possibility that the deformations of the water's surface might be correlated with motions relative to some other set of massive bodies is beside the point. For Newton's criticism of that definition requires him only to show that that the water's motion relative to its contiguous and immediately surrounding bodies – i.e. to the bucket itself - fails to correlate with the effects manifested in the water's surface. In criticizing Descartes's distinction between true and common motion, he needn't show that they could not be

correlated with the water's motion relative to any other set of bodies, say, for example, the stars.

3. The Two Balls experiment

Even granting that Newton is not guilty of the logical blunder attributed to him by Mach and his followers in connection with the bucket experiment, one might nonetheless wonder at this point whether there is any point in setting this particular record straight. After all, the bucket experiment is followed, two paragraphs later, by a very similar and almost equally famous thought experiment:

For example, if two balls, at a given distance from each other with a cord connecting them, were revolving about a common center of gravity, the endeavor of the balls to recede from the axis of motion could be known from the tension of the cord, and thus the quantity of circular motion could be computed. Then, if any equal forces were simultaneously impressed upon the alternative faces of the balls to increase or decrease their circular motion, the increase or decrease of the motion could be known from the increased or decreased tension of the cord, and thus, finally, it could be discovered which faces of the balls the forces would have to be impressed upon for a maximum increase in the motion, that is, which were the posterior faces, or the ones that are in the rear in a circular motion (Newton [1726] 1999, 414).

Given the similarities between Newton's bucket experiment and his two globes thought experiment, it might seem that filling out the Cartesian context of the bucket experiment merely saves Newton from the frying pan while leaving him in the fire.

Such an objection, however, would overlook two important points regarding Newton's thought experiment. The first, again, is what exactly the experiment is intended to establish. Significantly, Newton begins the paragraph containing the two globes experiment by explaining that although it is very difficult in practice to differentiate true from apparent motions, it is nonetheless not altogether hopeless. The grounds for Newton's cautious optimism are clear enough: He believes that *given* the theory of the *Principia*, it is possible to, among other things, distinguish true rotations from mere relative rotations. Indeed, in closing the scholium, Newton promises that "in what follows, a fuller explanation will be given of how to determine true motions from their causes, effects, and apparent differences, and, conversely, of how to determine from motions, whether true or apparent, their causes and effects," explaining that "this was the purpose for which I composed the following treatise." Taken in context, the two globes experiment can easily be seen for what it is, namely, an illustration of how the distinction crafted and defended in the first fourteen paragraphs of the scholium is to be applied in the main text of the *Principia*. To such an illustration one might reasonably object that it does not fit the theory proffered, or that in fitting the theory it shows its defects. What does not seem to be a trenchant criticism is the claim that one might be able to provide an alternative description of the experiment, under an as yet unformulated competing theory, beginning with different fundamental concepts and axioms.

The second important difference is that, in spite of their salient similarities, the set up of the two experiments is radically different. For the two globes thought experiment takes place at a level of idealization far removed from the bucket experiment. Where the latter attempted to indirectly support the dynamics of the *Principia*, the former draws out the implicit consequences of the theory going far beyond anything that Newton might claim to have witnessed himself. As such the two balls thought experiment is part of a hoary tradition of examining the logical consequences of empirical theories under impossibly idealized conditions. To such theoretical extrapolations one might raise any number of objections – as indeed Mach does. Such objections, however, will necessarily be different in kind from the simple *non sequitur* of which Newton has been accused, and, as Mach saw, would apply with equal success or failure to vast swaths of scientific practice.

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