Induction from a single instance: incomplete frames

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Abstract

In this paper we argue that an existing theory of concepts called *dynamic frame theory*, although not developed with that purpose in mind, provides an adequate and interesting framework for the discussion of the viability of certain compelling cases of induction from a single instance (while also allowing in a rather systematic manner certain other intuitively implausible cases of such induction to fail). The key role is played by the distinction that we introduce between complete and incomplete dynamic frames, for incomplete frames seem to be very elegant candidates for the format of the background knowledge used in induction from a single instance. We also provide some comments about the way incomplete frames can be accepted and their fallibility.

1 Induction from a single instance

The word 'induction' can refer to a whole variety of reasoning methods.[1] This paper however, is mostly concerned with a specific type of inference. Take the following two examples, motivated by (Norton 2003: 649) and (Steel 2008: 88) respectively:

$$\frac{\text{One sample of bismuth melts at 271 °C.}}{\text{All samples of bismuth melt at 271 °C.}}$$

$$\frac{\text{Bob's 2005 VW Beetle has it's wheel-drive in the front.}}{\text{All 2005 VW Beetles have their wheel-drive in the front.}}$$

These are inductions where from the fact that one element of a given class of objects has a certain property, a generalization is inferred which attributes

[1] Vickers (2009) gives a nice survey of those.

that property to all elements of this class. We will call this inference pattern 'induction from a single instance', and henceforth abbreviate it as ISI. The general schema for ISI is:

(ISI)

$$\frac{\text{One C has P.}}{\text{All C's have P.}}$$

where C is a predicate determining a class and P is a property.

Now, it is obvious that this schema doesn't hold for all classes and all predicates. Although examples of apparently reliable ISIs are numerous, it is just as easy to give a counterexample to this type of inference. If the category in the first example was wax, the induction would be unwarranted. It would be equally unreliable if the property in the second example was 'is blue' instead of 'has its wheel-drive in the front'.[2]

What is it that makes ISI rational in some cases, but not in others? This we will take to be an epistemological problem: we are not concerned with the metaphysical question of how come our inductions, predictions and hypotheses correspond to external reality, and we will not discuss ideas pertaining to the uniformity of nature. We are rather interested in an epistemological question: what background knowledge do we usually need to consider an application of ISI trustworthy and useful, what are our convictions when we disregard an application of ISI as misled, and what format could such knowledge be conveniently represented in?

There have been many approaches to such questions, and many different ways to justify and clarify the distinction between plausible and implausible cases of ISIs have been proposed. Notwithstanding the difference in concepts, most authors agree on the necessity of some specific background knowledge that accounts for the selective reliability of ISI. John Stuart Mill (1973: 308-311) calls it a 'hidden major premise' and pursues the idea that every induction is actually a syllogism . Goodman (1978: 110) speaks of a 'positive overhypothesis', Thagard and Nisbett (1982: 380) refer to 'knowledge of variability within kinds', Davies (1988: 233) points at knowledge of 'determination relations', and more recently Norton (2003: 650) refers to a certain kind of 'material facts'.

This background knowledge (BK) has to be strong enough to make a single instance sufficient for the inductive conclusion. However, BK shouldn't by itself

[2] The specific reasoning pattern that we call ISI is well-known in cognitive sciences and was shown to be a robust phenomenon in experimental settings. See e.g. (Thagard and Nisbett 1982: 380) for an example concerning the physical behavior of a kind of metal, floridium. Moreover, the impact of the projectability of certain properties on inductive inferences about these properties is a well-established fact in cognitive psychology – see (Heit 2000).

entail this generalization, for this would make the single instance redundant.[3] It is BK *together with* the instance that lets one to derive the desired generalization.

Take our second example. On the face of it, the background knowledge comes down to this: 'Either all 2005 VW Beetles have their wheel-drive in the front, they all have it in the rear, or they have a 4×4 wheel-drive. With this assumption, knowing that at least one car of this year and making has its engine in the front enables one to infer that all 2005 VW Beetles have their engine in the front.

This makes it clear what kind of BK is required for an ISI. That is, the BK should imply a disjunction of generalizations. Each of these generalizations should be about the same category C, assigning a certain property to all its members. That is, in this scenario we believe that one of these generalizations is true, and that all others are false,[4] but we don't know which one. The instance, itself an element of that category, falsifies all but one of these generalizations, and hence 'picks out' that generalization as the inductive conclusion. This turns the induction into a classical inference, which explains its great strength compared to other cases of ISIs which lack such additional assumptions.[5]

At this point, we suggest that every ISI depends on some domain-specific background knowledge which satisfies the formal criteria given above. Where this background knowledge is absent, inferences of the form of ISI are either very weakly justified, or not justified at all.

This is of course only a very rough sketch of an answer to the problem of induction from a single instance. More detailed questions now come to the foreground: How is this background knowledge related to our conceptual framework in general? Is there any uniform relation between the classes about which we generalize and the properties that we generalize about? How do we come to accept this BK? Is there any fairly structured way of obtaining it, and revising it when faced with anomalies?

Given that conceptual matters play an important part in the justification of any kind of induction, we may ask ourselves if a cognitive theory of concepts can suitably help frame the justification of a certain type of induction. Our claim is that the conceptual background that licenses ISI can be very conveniently studied with the use of the theory of *dynamic conceptual frames*. We will start with a brief explanation of what these are in section 2. Readers who are already

[3] Davies (1988: 231) in particular stresses this second point, and calls it "The non-redundancy problem".

[4] In section 6 we will discuss the possibility of BK itself turning out to be false.

[5] This doesn't mean that the assumptions used in this inference haven't been obtained by some sort of inductive reasoning. We'll discuss this possibility in section 5.

acquainted with Dynamic Frame Theory may skip this section. Section 3 introduces the distinction between complete and incomplete dynamic frames. It's one of the key distinctions needed in this paper. In section 4 we discuss the role this distinction plays in a fairly uniform account of ISIs. Section 5 is devoted to an explanation of how incomplete dynamic frames come to be accepted and developed. In section 6 we describe certain ways the frames may be revised when anomalies are encountered in the context of ISI. In the remaining section, we will discuss some arguments in favor of our approach, and remark on some loose ends and prospects for further research.

2 Dynamic frames

On the *classical theory of concepts* to each concept there corresponds a set of necessary and sufficient conditions for falling under that concept, a set of conditions that can be discovered by conceptual analysis. Arguably, the classical view is not an adequate picture of how concepts work in human cognition.[6]

One of the major and most recent accounts of concepts put forward as an alternative to the classical theory, inspired by the work of Rosch (1973b, 1983), employs the notion of a *dynamic conceptual frame*.[7] One of the most well-known formulations of the theory has been provided in (Barsalou 1987; Barsalou and Hale 1993; Barsalou 1993; Barsalou and Yeh 2006). Motivated by the work of Kuhn (esp. Kuhn 1974), certain applications to the history of science have been put forward and it has been argued that dynamic frames are a useful tool for accounting for scientific revolutions and conceptual frame

[6]It is not our purpose here to argue against the classical theory, but we will explain in a few words why one might want to reject it (the *locus classicus* is (Quine 1951), see however a more recent attack launched from a slightly different position (Fodor et al. 1999)). One of the problems is that the alleged definitional conditions of classical concepts are quite intractable: an average human being is usually unable to produce upon request a correct analysis (in the classical sense) of concepts that they use, and even philosophers — people who tend to spend an unusual amount of time on conceptual analysis — "have failed to provide a single generally agreed analysis of any important concept" (Andersen et al. 2006: 6). Another is that psychologists and cognitivists gathered psychological evidence to the effect that the structure of human concepts is graded — objects can fall under a concept to a higher or lower degree (Rosch 1973a, 1975a,b, 1978, 1983) — a possibility that can't be easily accounted for by the classical theory. Yet another worry arises when concepts are considered from a Wittgensteinian (1953) perspective. On this view, there are at least some concepts whose instances have no common features and bear a mere *family resemblance* to each other. In such a case any classical conceptual analysis of a concept is impossible because there are no necessary or sufficient conditions for falling under it.

[7]The psychological evidence for the adequacy of this theory is surveyed by Andersen et al. (2006: 47-52).

incommensurability (Andersen et al. 2006).

A frame developed for a single concept only is called a *partial* frame. In this paper we will be interested in various kinds of partial frames and certain relations between them. A partial dynamic frame (developed for a single concept A only) is composed of two layers of concepts: *attributes* and *values*. Every object that falls under A is supposed to have all the attributes. Objects having a certain attribute are divided according to what values of those attributes they instantiate. Take a really simple (and biologically inadequate) example (originates in Andersen et al. (2006: 43)). The concept BIRD can be considered in a frame where it has only two subordinate attributes: BEAK and FOOT, each having two values: ROUND, POINTED, and WEBBED, CLAWED respectively.[8]

Some combinations of values (from among each of the attributes one value is chosen) are considered to be *activated*. This means that objects that instantiate values from that combination are taken to constitute a separate *taxonomical unit*. For instance, in the exemplary frame there may seem to exist just two activation patterns: {POINTED, CLAWED} and {ROUND, WEBBED}, giving raise to the taxonomical units LAND BIRD and WATER BIRD respectively. In this sense, a (partial) dynamic frame specifies a taxonomy of the concept under consideration. The concept BIRD in the frame under discussion is divided exhaustively and exclusively into two separate taxonomical units.

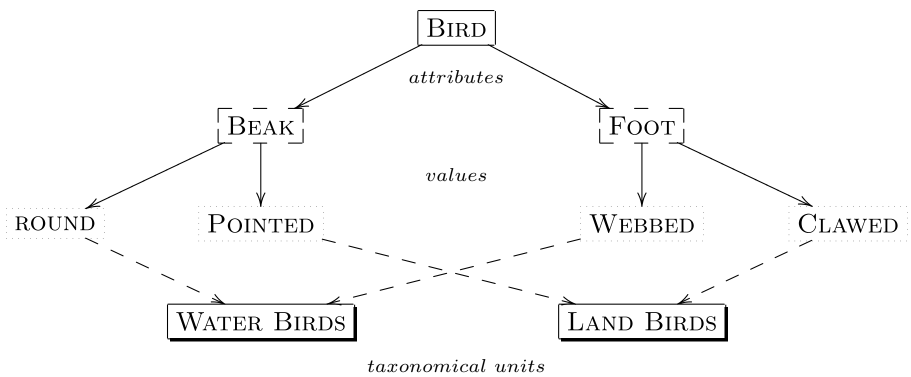

Figure 1: A partial dynamic frame for the concept BIRD.

So one of the main constituents of a dynamic frame is a *tree-like structure*.[9]

[8] We will refer to partial dynamic frames as *frames*, and a distinction soon will be made between complete and and incomplete frames. Completeness will be opposed to incompleteness, not to being partial.

[9] What are the *nodes* of a conceptual frame? The dynamic frame theory is more concerned with the structure of our conceptual framework rather than with the nature of its particular

The idea seems fairly simple. Any object that falls under the root concept is supposed to have one of the values for each of the attributes. Attributes are just aspects in which objects that fall under the root concept are classified and values are various relevant features that an object can have with respect to those aspects. This kind of structure provides frame theory with rich means of expression, and we will rely on this feature in subsequent sections.

Another important constituent of a dynamic frame are *activation patterns*. These decide which combinations of values for the attributes that occur in the frame actually occur together and constitute a separate taxonomical unit. For instance, in the frame from fig. 1, the combination {ROUND BEAK,WEBBED FOOT} constitutes a taxonomical unit of WATER BIRDS and the combination {POINTED BEAK,CLAWED FOOT} constitutes the taxonomical unit of LAND BIRDS.[10] On the other hand, the above frame does not admit an activation pattern where an object has a pointed beak but webbed feet, or a round beak and clawed feet.[11] We assume that if a taxonomical unit is determined by a certain activation pattern an object has to instantiate all the values from that activation pattern to be its member.

In a complete frame, for any object that falls under the root concept and for any attribute, this object has to instantiate exactly one value for that attribute.[12] Also, the taxonomical units that arise from the activation patterns are taken to be a division of the domain of objects that fall under the root concept: no object should belong to two taxonomical units (see Andersen et al. 2006: 56) and every object should belong to a taxonomical unit (Andersen et al. 2006: 27).

We have to be clear about remaining assumptions which are being made. On our level of analysis, we assume the *groundedness* of the whole frame, that is, that neither are attributes or values root concepts of further dynamic frames, nor are the nodes of a frame tree taxonomical units resulting from other frames.[13] We also assume that any partial frame is *finitary*: both the number of attributes

points. Barsalou (1993: 10) suggests that "frames are large collections of perceptual symbols," where perceptual symbols are supposed to be "aspects of experience stored in memory via selective attention that function symbolically" (Barsalou 1993: 5).

[10] The distinction between these two groups might be introduced for instance because there are certain useful generalizations that apply to all land birds but not to all water birds, and so on.

[11] This might be the case for instance because we have a causal story that tells us why birds with webbed feet are less likely to survive if they have a pointed beak, or because we simply have no evidence for there actually being birds instantiating this combination.

[12] "... all of the attribute nodes are activated for every subordinate concept. However, value nodes appear in mutually exclusive clusters." (Andersen et al. 2006: 44)

[13] In principle, the dynamic frame theory assumes that no level of analysis is ultimate, and that given a frame, each of its nodes can be further analyzed if need be.

and the number of values for each attribute are finite.[14]

3 Complete and incomplete Frames

Now that the notion of a dynamic frame has been briefly introduced, we will focus on a particular aspect of the theory, which we need to elaborate on in order to explain the relation between dynamic frames and induction from a single instance.

For some frames, each taxonomical unit has a fixed value for every attribute of the frame, i.e.:

> [**Strong Relevance Requirement**] For any taxonomical unit and for any attribute in that frame, there exists exactly one value for that attribute such that all objects in that taxonomical unit have that value.[15]

It is important to observe that SRR is different from the claim that for any object that falls under the root concept, for any attribute, there exists exactly one value for that attribute possessed by that object. For even if the latter condition holds, it still might be the case that every individual object has exactly one value for each of the attributes and yet a taxonomical unit contains objects that disagree on a certain attribute.

The basic intuition in support of SRR is that attributes should be, in a fairly strong sense, relevant for our classification of objects falling under the

[14]Some important aspects of frame theory are not being pursued here. For instance, we ignore the way how the frame theory is supposed to explain the graded structure of human concepts (the basic idea is that in one's experience, objects instantiating a specific activation pattern may have occurred more often than objects instantiating other activation patterns, and those objects are taken to be more typical representatives of the root concept (see Barsalou 1992: 47)). We also avoid getting into a discussion pertaining to the question whether the framework provides a good explanation of the phenomenon of *family resemblance* (claims that it does can be for instance found in (Andersen et al. 2006: 11). The basic idea is that a root concept is a family resemblance concept if no two activation patterns in its dynamic frame have a value in common).

[15]"Conventionally, all of the attribute nodes are activated for every subordinate concept. However, value nodes appear in mutually exclusive clusters. Only one value for any given attribute may be activated, but different activation patterns, or different choices of value, generate many different subordinate concepts, within the limits allowed by the attribute and value constraints already described. Each pattern of selection constitutes a subordinate concept; for example, a waterfowl is a bird whose values for BEAK and FOOT are restricted to ROUND and WEBBED." (Andersen et al. 2006: 44) We use the term "taxonomical unit" where Anderson et al. speak of subordinate concepts.

root concept. SRR is quite strong, because we also have another candidate for capturing the idea that attributes should be relevant for our classification:

> [**Weak Relevance Requirement**] For any taxonomical unit and for any attribute in that frame, there exists at least one value for that attribute such that no object in that taxonomical unit has that value.

We will focus on frames that obey SRR, arguing that these provide the most straightforward justification of ISI. However, in section 7 we will briefly show that ISI can take place in the context of other frames as well. In other words, the rather strict and abstract model we are presenting can be easily loosened such that real-life examples are within reach.

Now, even if SRR holds, a distinction should be made between the case where we believe that every taxonomical unit has a fixed value for each attribute, but are sometimes unable to tell which value a given taxonomical unit is associated with, and another case, where we not only accept SRR, but also directly associate each taxonomical unit with a single value for each attribute. These are epistemically different situations, and – as the examples we will give indicate – both cases are quite common.[16] If all values of each taxonomical unit within a frame are known, we will speak of a *complete frame*. If not, we will speak of an *incomplete frame*. Since they are less specific, incomplete frames are less informative than their complete counterparts. In this sense, when we "fill in" the details in an incomplete frame, associating taxonomical units with specific values, epistemically speaking, we are making progress. In what follows, we will describe induction from a single instance as (a part of) the process of rendering an incomplete frame complete.

Take for example the concept BROADLEAF TREE. Suppose the attributes of the frame for this concept are LEAF-SHAPE, FRUIT and MODE OF REPRODUCTION.[17] Suppose we know three kinds of broadleaf trees: chestnut trees, elms and cherry trees. These are our taxonomical units. We may once have learned the shape of the leaves for each of these units. However, few of us still know the leaf-shape of, say, elm trees. We know that all elms must have the same kind of leaves, but we can't figure out what exactly these look like, and we may not be able to choose the right item from a list of all possible leaf-shapes. Hence, our frame for broadleaf trees is incomplete (figure 2):

[16] As far as we know, the distinction has not been made in literature of the subject.

[17] For the sake of brevity and simplicity we use a greatly simplified example here, in the sense that we've reduced the number of attributes, values and taxonomical units to a minimum.

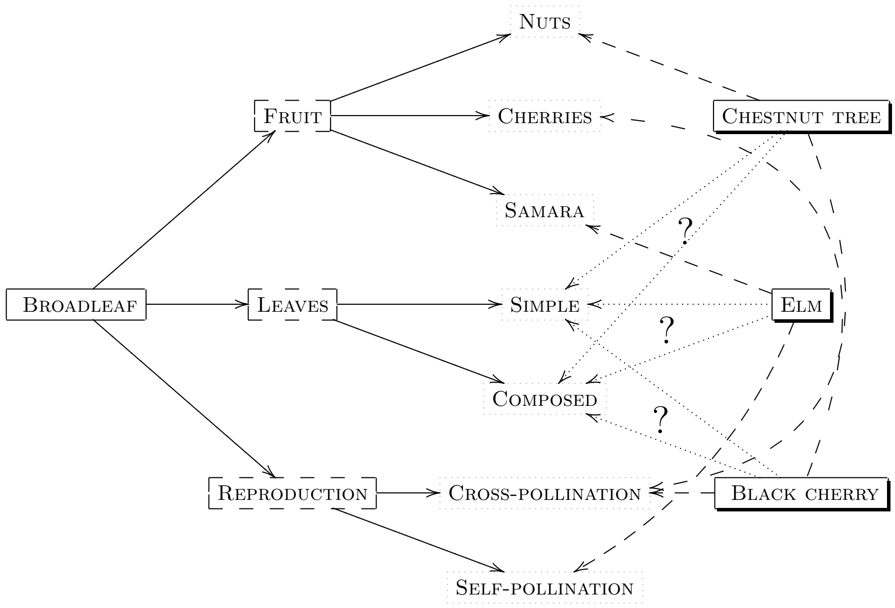

Figure 2: An incomplete frame for BROADLEAF TREE.

A tree expert, on the contrary, may have a complete frame for broadleaf trees (figure 3):

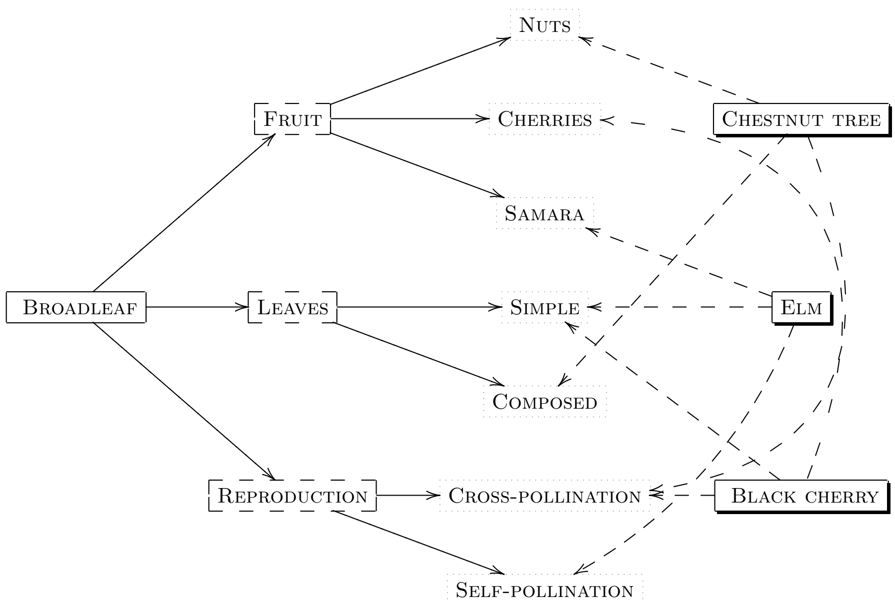

Figure 3: A complete frame for BROADLEAF TREE.

The incomplete frame presented in figure 2, even though less informative than the one pictured in figure 3, is still in an important sense more informative than a frame without LEAF-SHAPE as an attribute. In the latter case, we wouldn't even have the incomplete knowledge about taxonomical units being uniform with respect to this attribute. This indicates that, despite its name, an incomplete frame can actually be quite informative.

Moreover, given that it might be rather hard to memorize all the shapes of leaves of different tree species, and that we often can quite easily consult exemplars of trees in our environment and obtain this information, the incomplete frame may be seen as a "shorthand guide" to certain information.[18]

4 Incomplete frames and single instance induction

What is the relation between incomplete frames and ISI, as we described it in the first section? The answer is fairly straightforward: incomplete frames may be seen as providing the background knowledge necessary for ISI. The property that the generalization assigns to certain objects is a value of an attribute of such a frame, and the class of objects that the generalization is about is a taxonomical unit in that frame. Each ISI which draws a connection between a particular taxonomical unit and a certain value is a step towards a complete frame.[19] It is the presence of an incomplete frame that turns ISI into a reasonable inference. To get a better grip on this approach, let's reconsider the examples from section one.

Assume that we have an incomplete frame for PURE LIQUIDS, containing the attribute BOILING POINT. Since BISMUTH is a taxonomical unit in this frame, all samples of this fluid will have the same boiling point. Knowing the boiling point of one sample of bismuth thus suffices to associate the right value for the attribute BOILING POINT to the taxonomical unit. (Melted) wax, on the contrary, is not a taxonomical unit of this frame (or any other incomplete frame with BOILING POINT as the attribute, for that matter). Hence a similar ISI doesn't work for wax.[20]

As for the car example, say an incomplete frame CAR contains an attribute

[18]One may compare this to information gathering in general: it is sometimes more useful and certainly easier to know where you can find the right information about a certain class of subjects when you need it, than to know as much as possible.

[19]Whenever a series of ISIs eventually leads to a complete frame, one can speak of the "saturation" of an incomplete frame.

[20]Notice that the expression "has a fixed boiling point" can be captured within Frame Theory only given the distinction between complete and incomplete frames we made in the previous section.

WHEEL-DRIVE (2-wheel-front, 2-wheel-rear, 4-wheel). The taxonomical units are version-specific makes of cars. Our incomplete frame expresses – among other things – our knowledge that cars of the same version-specific makes have the same wheel-drive, and this licenses an ISI: once we observe that a certain car of a particular version has a certain kind of wheel-drive, we can legitimately infer that all cars of that version have the same kind of wheel-drive. On the other hand, COLOR is not a relevant attribute of this frame, since cars of the same type can have different colors .[21] The richness of Frame Theory, which for each concept introduces two levels of properties (attributes and their values), makes it a suitable framework for an account of ISI. All the different constituents of a frame play their part in the explanation.

5 Obtaining incomplete frames

Once we have an incomplete frame, it assists us with sensible inductions from a single instance. Also, inductions from a single instance not sanctioned by an available incomplete frame are not legitimate, and in this sense our assembly of available incomplete frames allows us to draw the line between convincing and unconvincing ISIs.

How do we reach a stage where we have an incomplete frame available, though? Our suggestion is that a complete frame which is to be extended into an incomplete frame provides us with a frame of reference for certain second order inferences. How is that supposed to work?

Say we have a complete frame with a certain taxonomy available with taxonomical units $T_1, T_2, \ldots, T_n$ available. Further reasons may convince us that there is a certain property P (our candidate for an attribute) associated with a certain assembly of properties $V_1, \ldots, V_k$ such that every object that falls under our root concept is such that if it has P, it has exactly one of the properties from among $V_1, \ldots, V_k$.

One way this can lead to an incomplete frame, is that we may be informed by a causal or conceptual story which convinces us that each taxonomical unit is associated with a single value for that attribute P. By then, we come to believe that each T_i is associated with a single value V_k.

For instance, suppose we're developing a frame for our concept CAR. We already came to believe that the basic taxonomical units are version-specific car makes. Among our attributes, for instance, we have the number of doors, or the

[21]Note that, since no colors are excluded for any type of car, even a frame for CAR that only conforms to the Weak Relevance Rule cannot contain the attribute COLOR.

size of the engine. We come to believe that all types of cars (thus understood) have a certain wheel-drive, and we come to distinguish between a car's being a 2-wheel-front-drive, a 2-wheel-rear-drive or a 4-wheel-drive.[22] Suppose that even though our criteria for belonging to a certain version-specific car make did not employ the notion of wheel-drive, we also become convinced that the producers treat cars with different drive-wheels as constituting different versions. So we come to accept the conclusion that every version-specific car make is associated with a single value for the attribute WHEEL-DRIVE, even though we are not sure what the connections exactly are.

Another scenario takes place if we consider a certain induction on a meta-level. Here we obtain an incomplete frame by generalization. So, given our complete frame, we come to believe that T_1 is associated with V_1, T_2 is associated with V_2, and in general we come to accept a certain number of generalizations of this sort. Then, prior to discovering those connections for all of the taxonomical units, we reason by induction *on the taxonomical units themselves* to the conclusion that all the taxonomical units available in that frame are associated with a single value, thus obtaining an incomplete frame.[23] This, of course, may motivate our search for a causal or conceptual story explaining this connection, but that's quite a different issue.

For instance, suppose we're developing a frame for the concept *pure liquids*. Our attribute candidate is BOILING POINT. We learn that pure liquids of a certain type all have the same boiling point. Say we learn that about quite a few types of pure liquids, but we still don't have any causal story available that would convince us that all types of pure liquids are connected with fixed boiling points. We still might be in a position to infer inductively that each taxonomical unit in our frame is associated with a fixed boiling point.

We tend to be pluralists about the ways an incomplete frame can be obtained, and the above is not a complete list of the ways this can be done. Just like there are many ways one can be justified or rational in accepting certain beliefs, there are many ways one can be rational to accept a certain structured conceptual frame. Rather our intention was to display at least certain ways an incomplete frame can be obtained, to indicate that this seems like a fairly natural and

[22]We acknowledge the existence of 6WD and 8WD vehicles; we just don't count them as cars.

[23]We say this reasoning is "in a sense" second-order, because it's an inference to a generalization about taxonomical units, not about objects themselves. However, as far as finite frames are involved, the reasoning (and the generalizations) can be represented in terms of a first-order language and first-order inference rules. More on formal representation of dynamic frames in (Urbaniak 2009).

common process.[24]

6 Revising frames

While reasoning with a certain (complete or incomplete) frame in the background, reliable data can be encountered which might force one to revise the frame.[25] Roughly speaking, it might turn out that the taxonomization provided by the frame is inadequate. For instance, an object falling under the root concept can be discovered which does not belong to any of the taxonomical units (or which should belong to two distinct taxonomical units). There are quite a few ways in which data may go against a given frame. In the current context, however, we are interested in those cases, in which the beliefs introduced during the process of obtaining an incomplete frame and those that are indirects result of this introduction – the generalizations that result from ISIs – are undermined.

Suppose we have filled in the blanks in an incomplete frame, by drawing inductions from a single instance. Every taxonomical unit T_i is thus associated with a single value for the new attribute A, say V_j. We happily continue to use the newly obtained complete frame for a while.

Alas, at some point we realize that two objects that fall under the same taxonomical unit have a different value for A. So we are faced with a contradiction between the (formulas describing the) newly obtained complete frame and the (formulas expressing) data available. Of course, for a while we can weaken our reasoning principles and reason non-classically in all those cases where the problematic part of our framework is involved.[26] This, however, is only a provisional way to handle the situation. Having inconsistent beliefs usually is not

[24] In line with our remark in footnote 28, we would like to emphasize that the picture of incomplete frames being obtained from complete frames does not assume that our conceptual framework starts with complete frames. Rather, the process of obtaining an incomplete frame from an incomplete frame is quite similar to the one we discussed and since it doesn't seem to bring in any serious complications, we've decided to just point out that it's *mutatis mutandis* the same.

[25] For more details on formal aspects of such situations and a discussion of the logical apparatus used temporarily during frame revision see (Urbaniak 2009).

[26] In classical logic a principle holds according to which anything follows from a contradiction. Numerous logics weaker than classical logic have been developed that invalidate this principle and allow for (more or less) sensible reasoning with contradictory data. These are called *paraconsistent logics* (see for example: Priest 1979; Priest and Routley 1983; Priest 1987, 2008). Another important group of logical systems which (to some extent) has been developed to handle inconsistent data are *adaptive logics* (see for instance: Batens 1995, 1999; Meheus 1999, 2000; Batens and Haesaert 2003). See (Urbaniak 2009) for more details regarding the relation between these logics and dynamic frames.

the ideal state that we are after and we need to revise our frame somehow.

Our Strong Relevance Rule suggests that either T_k should not be treated as a taxonomical unit, or that A and it's values should not be included in the frame. In this sense, one way to revise the frame is to revert it to the initial complete frame in which A is not present.

Perhaps, even though we have reasons to be suspicious about our generalization about T_k, our claim that taxonomical units are associated with specific values is still strongly supported for all the other taxonomical units. In such a case, we would be rather inclined to keep the new attribute, and divide T_k into as many "real" taxonomical units as needed: if all the objects from T_k have one of the values V_i, V_n or V_u, we might simply move to a frame where instead of T_k we have three taxonomical units all of them associated with the same values for all the attributes different from A, and each of them associated with a different value for A. For instance, in the case of wheel-drives, once we discover that something that we considered a specific version of a car comes in two sorts: those that have 2-wheel-front and those that have 2-wheel-rear drive, instead of deleting the attribute of wheel-drive, we might just decide that what we considered a version is not really a version and comprises at least two versions.

One may also decide to split up the original frame into two separate frames for two different root concepts, one containing the new attribute, and the other one without it. This implies that one also decides which elements of $T_1, T_2 \ldots, T_k$, belong to the first frame, and which to the second one. For instance, one may initially have an incomplete frame for SUBSTANCES, containing the attribute BOILING POINT. After discovering that wax doesn't have a fixed boiling point, one may distinguish between pure substances and mixtures. The frame for PURE SUBSTANCES still contains the attribute BOILING POINT and remains an incomplete frame, while the one for MIXTURES doesn't. A good reason to do this would be that one knows of other relevant differences between both subcategories of the initial root concept (as is the case for the example we just gave).

What matters to us here is that there are many rational ways one can respond to inconsistencies that result from new data. The exact details of these reactions require further attention, and perhaps more formal accounts. Our goal was to indicate that Frame Theory can provide us with the theoretic tools to do this.

7 Final remarks

It might seem that the framework of incomplete frames is not versatile enough to model non-ISI kinds of induction. Here are some examples of worries that one might have:

- Given that we conclude that melted wax isn't a taxonomical unit having a fixed melting point, how can we model the information that every piece of wax has a melting point?
- The system might not seem to deal with knowledge that fixed boiling points depend on other circumstances, like air pressure.
- Color is not an attribute of the frame for cars, because cars of the same taxonomic unit can have different colors. Thus, we have no way to represent the fact that sports cars tend to be brighter colors than non-sporty cars (similarly that sporty cars' tires will tend to be wider and flatter than non-sporty cars' since the specific values vary among the sporty cars), nor apparently to realize that every car has to have some color.
- Ford Explorers come in both real-wheel and four-wheel drive – are they really "different taxonomic units"? And, particularly, are they really contrasted at the same level? Or, are they two different sub-types of Ford Explorer? (which would mean that taxonomic units don't actually have to have only 1 value of the attribute, so the attribute is no longer part of the frame?).

What each particular partial frame might miss can be within this framework made up for by the abundance and contextual sensitivity of conceptual frames. So in the first case, we nevertheless can simultaneously have a wider frame for the root concept 'material object' (or what have you) and 'the temperature at which all its parts will melt' as an attribute. The second worry in the third case can also be dealt with in an analogous manner. In the second case, it is rather clear that what is meant by fixed boiling point is relative to salient standard conditions (and if one wishes, there's no obstacle to introducing separate fixed boiling points for as many different relevant circumstances as one wishes). The first part of the third case is a bit more complicated, but again, there is no reason why one couldn't simultaneously have a different frame for the same root concept 'car' where 'being sporty' is one of a values for one attribute, and color-mood a value for another (clearly, one and the same class of objects can be taxonomized differently for different purposes). The fourth case turns on the (implausible) assumption that for any root concept we might have at most one partial frame. In fact, it's possible that insofar as technical classification of cars is needed, these two types of Ford Explorer in fact are two different taxonomical units in a frame in which drive is an attribute, but that simultaneously, insofar as every-day notion of car brands go, they belong to one taxonomical unit which doesn't contain such an attribute.

A related, more general concern might be that we introduce the notion of an incomplete frame just to make the dynamic frame theory capable of modelling ISIs (for standard induction reasoning, it seems, doesn't fit the pattern we discussed), and that this move seems quite *ad hoc* and goes against the uniformity of dynamic frame theory.

Whether the move is *ad hoc* depends not on whether incomplete frames model other kinds of reasoning, but rather on whether we have independent reasons to think that we sometimes operate with incomplete frames. And our suggestion is that yes, we do. Even now, I know that every kind of tree has a specific sort of leaves and yet I often have no idea about the details. Whether the move disturbs the uniformity of the original theory depends on a rather vague notion of uniformity, but it seems that for any induction relies on the grouping of predicates by attributes (in Goodman's words: on a meta-reasoning about the projectability of predicates), which is the key move we employed.[27]

In the previous sections, we explained how Mill's problem concerning the appropriateness of single instance inductions can be approached in a fairly clear way in terms of the existing theory of *dynamic conceptual frames*. Only a few specifications and one distinction – that between complete and incomplete frames – suffice to phrase the necessary BK for ISI in terms of a partial dynamic frame. Furthermore, questions concerning the acceptance of this BK and the reaction when confronted with data that contradict it can be addressed within this framework.

The concept of an incomplete frame thus helps us to formulate the necessary background knowledge for ISI, bringing with it all the theoretic tools that are inherent in Frame Theory. Interestingly, Frame Theory was not constructed with ISI in mind, and its development by cognitive scientists is quite independent of questions concerning induction. In this sense, developing Frame Theory to model ISI does not seem to be too *ad hoc*.[28]

[27]More details on the relation between ISIs and other kinds of induction as seen from the perspective of dynamic frame theory have to wait for another paper, though.

[28] Although complete frames, epistemically speaking, might be more desirable because they contain more information, we make no claim about the "primitivity" of complete frames. We are not saying that complete frames are in any sense more basic or common in human

The most striking feature of Frame Theory, compared to preceding alternatives to the classical view of concepts, is that it allows for two levels of properties: attributes and values. Hence it makes it possible to group values according to the respective attributes that they are values of. Whether the BK is obtained through causal knowledge or through a kind of second order induction about properties and classes of objects, there is always a reference to a set of properties, namely all the values of a certain attribute. Most of the older alternatives – for instance the prototype theory (Rosch 1973b) or the exemplar theory (Medin and Schaffer 1978) – lack the division of properties into levels which would allow for this sort of move.[29]

What may be slightly unusual about our approach is that we consider the appropriateness of ISIs to be a matter of concepts. However, we are not the first to state that a conceptual framework contains more than just analytic knowledge in the sense of possible combinations of values that fall under a root concept, but can contain factual and law-like knowledge as well.[30] The notion of complete and incomplete frames is simply an extension of this idea.

We promised to come back to the possibility of frames that only obey the weak relevance requirement. Even for such frames it is still possible that at least one attribute behaves according to the strong relevance requirement (even though the whole frame doesn't), and yet we don't know which taxonomical unit is associated with which value. The justification for the assumed behavior of such an attribute, is identical to the one we gave in section 5 and ISI proceeds in the same manner as before.

Numerous issues still require consideration. Further research should include both a more formal account of the acceptance, use and rejection of incomplete frames, continuing the work initiated by Urbaniak (2009) and more psychological research pertaining to the empirical adequacy of frame theory. The relation between ISI and other kinds of induction within the context of frame theory also deserves some consideration. We have already pointed at the possibility to make weaker sorts of inductions, once an incomplete frame is contradicted by our data. This is just the starting point of an investigation into the relation between frames and induction as a whole. Following Nelson Goodman's critique

cognition. Quite to the contrary, it seems that our background knowledge quite often should be rather represented in form of an incomplete frame (perhaps, other kinds of incompleteness can be considered).

[29] See Barsalou and Hale (1993) for a lengthy discussion of this difference.

[30] This is stressed in particular by the followers of the so-called Theory-Theory view. See Andersen et al. (2006: 60-64) where this view is commented on. More generally, activation patterns and structural invariants already go beyond what we would be inclined to treat as "analytical knowledge".

of a purely syntactical approach to induction,[31] Frame Theory can provide a semantic complement to logics of inductive generalization and thus viewed, may consist a step toward a more general theory of inductive generalizations.[32] The general idea behind this is that our inductive reasoning can only reach rather stable and reliable universal propositions once it limits its scope to a certain set of predicates – those that are values of attributes in a frame. Such a model of induction would be able to model both ISI and more complex cases of induction.

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[31]See Goodman (1978), where the author presents his famous “Grue paradox”.

[32]See Batens and Haesaert (2003) for examples of a specific class of logics of induction that we have in mind, adaptive logics of induction. These are being developed by the Ghent Group.

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