Theory Change as Dimensional Change.

Conceptual Spaces Applied to the Dynamics of Empirical Theories

Peter Gärdenfors & Frank Zenker (peter.gardenfors@lucs.lu.se, frank.zenker@fil.lu.se)

Department of Philosophy, Lund University

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Abstract: It is suggested that the dynamics of scientific knowledge may be reconstructed in terms of structured collections of dimensions – conceptual spaces – underlying empirical theories. Five change operations are identified by which to rationally reconstruct dimensional change. Given this classification, the conceptual development of empirical theories appears more gradual than what Kuhn depicted. Only the most severe type – replacement of dimensions – comes close to a Kuhnian revolution.

The five types are exemplified and applied in a case study. The proposed approach is Neo-Kantian. In contrast to the approaches of Thomas Kuhn and Michael Friedman, however, the reconstruction of the dynamics of scientific knowledge may be achieved without a commitment to some dimension(s) being methodologically *a priori*.

Keywords: Michael Friedman, Thomas Kuhn, conceptual spaces, dimensional analysis, incommensurability, methodological a priori, scientific revolution

1. Introduction

During the heydays of logical empiricism, an empirical theory was considered to be a set of sentences – laws and others – and describing scientific change was mainly a question of how new sentences could be added. Science was in general seen to be cumulative. In reaction to this conception – and echoing Ludwig Fleck (1935/1979) who took *crisis* to be a third developmental stage –, Thomas Kuhn (1962/1970) presents his account of scientific change. It consists of only two types: *revolutions* as dramatic shifts of paradigms with periods of *normal science* in between.

For Kuhn, the historical development of scientific knowledge is not cumulative. On the contrary, old and new paradigms are considered mutually incommensurable: acceptable problem-solutions, methods, meanings of crucial terms, and scientists' 'world-views' may change. His main example comes from modern physics. It concerns the difference in meaning of the term *mass* in Newtonian mechanics and in Einstein's relativity theory. The example serves Kuhn to evidence revolutionary change by relating the sentential representations of historically successive frameworks, i.e., special laws and their axiomatic basis.

We will argue that Kuhn's dichotomy is too simplistic as a general account of the dynamics of empirical theories. Rather than scientific laws, we focus on the conceptual frames. The frames will be modeled in terms of *conceptual spaces* (Gärdenfors 2000). The focus is on the dimensions underlying a theory and their structure. Our main point is that many types of scientific change can be understood as modifications of the conceptual frames involved.

Our approach allows us to describe these changes on a more fine-grained scale. We present five types of changes in section 4: (1) Addition or deletion of special laws (which does not involve any conceptual change); (2) change of scale or metric; (3) change in importance of dimensions; (4) change in independence of dimensions; and (5) addition or deletion of dimension. Given this classification, the conceptual development of empirical theories appears more *gradual* than what Kuhn depicted. Only the most severe type – replacement of dimensions – comes close to a Kuhnian revolution.

As a case study, we present in section 5 some of the conceptual changes that took place within Newtonian mechanics before the introduction of special relativity. We argue that much of the conceptual structure of relativity theory had been prepared in the development of mechanics. There *was* a radical shift in the conceptual frame of special relativity, but it is not as revolutionary as Kuhn and his followers claim.

Our analysis of scientific change can be viewed as a version of Neo-Kantianism in philosophy of science. We conclude with a discussion of some of the epistemological consequences of our position, comparing it to the Neo-Kantianism of Michael Friedman and Kuhn himself.

2. Kuhn's Sentential-Ontological View of Frameworks

2.1 The Revolutionary-Normal Dichotomy and its Problems

In *The Structure of Scientific Revolution*, Kuhn (1962/1970) tied the severity of theory change to the severity of its sentential reconstruction. He identified revolutions with meaning changes, a sentential representation of frameworks serving as the evidence base. On Kuhn's view, historically successive frameworks save phenomena under different conceptual contents. With reference to Einsteinian concepts applied to comparatively small (but not too small) masses at comparatively low velocities, he correctly observed that:

"The variables and parameters that in the Einsteinian E_i's represented spatial position, time, mass, etc. still occur in the [Newtonian] N_i's and they still represent Einsteinian space, time and, mass. But the physical referents of these Einsteinian concepts are by no means identical with those of the Newtonian concepts that bear the same name. (Newtonian mass is conserved; Einsteinian is convertible with energy. Only at low relative velocities may the two be measured in the same way, and even then they must not be conceived to be the same.)" (Kuhn 1970, 101f.)

His thesis of a world-change *qua* meaning change is *prima facie* supported by this observation, and so is his estimate of how revolutionary changes bear significance, namely *as wholes*.

“Revolutionary changes are somehow holistic. They cannot, that is, be made piecemeal, one step at a time, and they thus contrast with normal or cumulative changes like, for example, the discovery of Boyle’s law. In normal change, one simply revises or adds a single generalization, all others remaining the same. In revolutionary change one must either live with incoherence or else revise a number of interrelated generalizations together. If these same changes were introduced one at a time, there would be no intermediate resting place. Only the initial and final sets of generalizations provide a coherent account of nature.” (Kuhn 1987, 19)

In our opinion, the crudeness of two basic types of scientific change results from Kuhn’s focusing exclusively on a theory’s sentential forms. It hardly surprises that he finds that changes which amount to more than the mere addition (of a special law) cannot result in an “intermediate resting place.” Similarly, a non-revolutionary development of scientific knowledge, for Kuhn, will always have to be represented as an addition to an otherwise preserved set of sentences.

This characterization of change rather misleads. If a more fine grained “dynamics of dimensions” is acknowledged as an alternative model for conceptual change, then “revision of (a number of) interrelated generalizations” no longer characterizes only a scientific revolution. And normal science – which, after all, is *cumulative* – no longer demands stable parts of a framework to provide the “resting place,” whatever that may be. At the same time, the long term guiding-effect which frameworks provide in seeking cooperation from nature can be acknowledged.

By ‘framework,’ we designate what structuralists call ‘theory-core,’ but refrain from identifying an empirical theory *via* its core. The identity of a framework is normally given by a *theory core* or a set of *frame conditions*, e.g., $F = ma$ for Newtonian Mechanics. A new core is logically inconsistent with its predecessor.[1] This mode of fixing the identity of theories, we argue, creates the incommensurability issue. We rather seek to identify a new framework through systematic change operations applied to a predecessor framework.

2.2 Limiting Case Reduction does not fully address Kuhn’s Gap

When comparing historically successive frameworks, the later framework tends to relate to its predecessor. Frameworks “share content” in the special sense of limiting case reduction. Reduction starts from a successor frame F^* and demonstrates mathematical continuity with a frame predecessor frame F (Batterman 2003; Nickles 1973; Post 1973).

As an example, consider *momentum*. The special relativity (SR) form $p = m_0 v / \sqrt{(1 - (v/c)^2)}$ converges to the Newtonian mechanics (NM) form $p = mv$ as $(v/c)^2$ goes to zero.

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1 Kuhn seems to have adopted the idea from Lakatos (1978), so did the structuralists (Balzer et al. 1984, 1987, 2000; Gähde 2002; Moulines 2002; Sneed 1971; Stegmüller 1976). His reservations against the structuralist view are expressed in Kuhn (1976).

Kuhn's claim is simply that m_0 and m amount to different definitions of mass.[2] In plain words: Yes, the SR-term $\sqrt{(1 - (v/c)^2)}$ approaches 1. However, the NM-form never contained it. Kuhn takes this as evidence for a meaning difference between frameworks.[3] Generally, the sentential form *S** of a successor framework *F** that is reached by limiting the value-range of some parameter in *S**, although at that range effectively indiscernible from the predecessor form *S*, need not therefore be identified with *S*.

For Kuhn, *severe* conceptual change is evidenced by the very meaning differences which limiting case reduction bridges. These differences came to be called "ruptures," and the ruptures came to indicate "rationality gaps" (Rehg 2009, 33-80). The (suggestive) image is that of a *non-smooth* change from framework *F* to *F**.

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[2] Expert-disagreement over the status of m_0 and m persists. See Rivadulla (2004) for the meaning-constancy standpoint or Falkenburg (2007, 161ff.) for the meaning-divergence standpoint. In the formula above, v is the velocity and c the speed of light in a vacuum.

[3] As a second example, consider that Euclidian space is a limiting case of Riemannian space when the latter's inner product is restricted to positive values. Scheibe (1999, 88) comments on a similar (though more complex) recovery of Newtonian mechanics from general relativity: "Even if, in the end [five pages !], the Newtonian field equation … has appeared, *with respect to the metric* …, we stand in the midst of the general theory of relativity" (*italics added,* our translation). In brief, Scheibe distinguishes 'local absence of mathematical difference' from 'semantic identity.'

The problem (same linguistic form, different conceptual content) suggests that Kuhn's gap separates sentences; Kuhn's rejection of its standard treatment (limiting case reduction[4]) suggests that the problem also pertains to *ontology*. This at least seems to be Kuhn's mature position. He came to deny a principled non-communicability across frameworks and regretted having earlier likened 'world view-changes' to Gestalt-shifts (Kuhn 2000; Larvor 2003). Using logical inconsistency to characterize the relation between incommensurable theory pairs – compare the structuralist's theory cores and the idea of a core-rejection to "model" a revolution –, the phrase 'incommensurability of frameworks' came to characterize general differences in methods and meanings (Oberheim and Hoyningen-Huene 2009).

We seek to model meaning difference between conceptual frames, but take issue with the claim that it defies a finer grained reconstruction of theory dynamics. For this purpose, we draw on *conceptual spaces* (Gärdenfors 2000). The proposal is to focus not on sentential representation (axioms and laws), but on the underlying dimensions, i.e., to model meaning change as dimensional change. As argued below, at this dimensional level, frameworks are fully comparable. *Pace* ontological considerations, therefore, the incommensurability of scientific theories modeled as sets of sentences poses no special problem for modeling

[4] Reductive attempts *via* bridge principles in the style of Nagel (1961) face well-known problems. Bridge principles always add semantic content. Thus, not the predecessor, but some improved version is "reduced" (Batterman 2003, ch. 5).

conceptual change. We propose that modeling scientific changes on the conceptual level is more appropriate than using sentential forms.[5]

3. Modeling Theory Change in Conceptual Spaces

3.1 Separable and Integral Dimensions

We next present conceptual spaces as a meta-framework by means of which theory-frameworks can be reconstituted. The basic components of a conceptual space are quality dimensions. The notion of a dimension should be understood literally. It is assumed that each quality dimension is endowed with certain *geometrical* structures.[6]

Psychological examples of such dimensions connected to sensory impression are *color*, *pitch*, *temperature*, *weight*, and the three ordinary *spatial dimensions*. However, in scientific theories the dimensions are determined by the variables presumed in a theory. To illustrate the scientific interpretation of 'dimension,' consider *mass* [M], *length* [L] and *time* [T], as used in

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[5] This is in line with Gärdenfors' (2000) arguments for distinguishing between the symbolic and conceptual levels in knowledge representation.

[6] In some cases they are *topological* or *orderings*. Points in dimensional spaces stand for objects or individuals, regions for properties and relations. Exploiting distances between them, *degrees of similarity* between objects can be modeled.

Newtonian mechanics. The first two dimensions have a zero point, and are thus isomorphic to the half-line of the non-negative numbers, while *time* is isomorphic to the full real line.[7]

The dimensions are taken to be independent of symbolic representations, in the sense that we can represent the qualities of objects (e.g., by vectors) without presuming an explicit object language in which these qualities are expressed. Conceptual spaces is presented as a framework for representing empirical theories that is different from both the symbolic approach, where everything is supposed to be expressed in sentential form, and Kuhn's analysis that builds on the holistic notion of a paradigm, combining conceptual and ontological considerations in an unfortunate way.

Dimensions are sorted into *domains*. In psychological applications, these can be obtained by distinguishing between *integral* and *separable* dimensions (Melara 1992, Maddox 1992). For example, an object cannot be given a hue without also giving it a brightness value. Or the pitch of a sound always goes along with its loudness. Dimensions that are not integral are said to be *separable*, as for example the *size* and *hue* dimensions.

Within the context of scientific theories, the distinction should rather be defined in terms of *invariance transformations*. For example, the three dimensions of ordinary Euclidian *space* (*x*, *y*, *z*) are separable from *t* (the time coordinate) under Galilean transformations (as in Newtonian mechanics), but not under Lorentz transformations (as in special relativity).

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7 We prefer 'isomorphic to' over 'homomorphically embedded in,' since infinitesimally small differences (e.g., in the lengths of objects) fall below the thresholds of cognitive and measurement capacity. Yet, they *are* part of a conceptual space. See Batitsky (2000, 96).

Moreover, *mass* is separable from everything else in Newton's theory, but not from *energy* in special relativity. It is part of the meaning of "integral dimensions" that they share a metric. The role of the invariance transformation classes in the specification of a theory's conceptual space will become clearer in the case study in section 5.

A *domain* of a theory can now be defined as the set of integral dimensions that are separable from all other dimensions. More precisely, domains *C* and *D* are separable in a theory, if the invariance transformations of the dimensions in *C* do not involve dimensions from *D*. This criterion for identifying a domain is tightly connected to the respective measurement procedures. For example, in classical mechanics, the measurement of distance and duration (trigonometry and chronometry) are independent. Light signals are tacitly assumed to propagate instantaneously rather than at finite speed. Likewise, the mass of an object is presumed to be independent of that object's position or velocity. Yet note that *heat* and *work* had been considered separable until the definition of heat as mean kinetic energy established that one can be measured in terms of the other. We return to this below.

3.2 Discriminate Dimensional Analysis

In reconstructing empirical theories via their underlying dimensions, we can compare with *dimensional analysis* (Bridgeman 1922, Huntley 1952, Palmer 2008, Raleigh 1915). Our proposal is to model theoretical frameworks as *structured collections of dimensions*. For example, when adding to 3D Euclidian *space* [L^3], the 1D *time* [T], the 1D *mass* [M], and three integral dimensions of *force* $[F^3]$ – isomorphic to a three-dimensional Euclidian (vector) space –, one reaches the conceptual space of the original Newtonian Mechanics: It is an 8D space with four domains: $[L^3]$, [T], [M], $[F^3]$.[8] So-called "derived magnitudes" (e.g., *velocity* $[LT^{-1}]$ or *acceleration* $[LT^{-2}]$), simply connect domains without affecting the independence of the respective measurement procedures.

A standard objection against representing scientific concepts in terms of dimensions is that distinctions are lost between vector and scalar quantities. For example, *torque* and *energy* would both take the form $[ML^2T^{-2}]$. Following Coulson et al. (2007, 20f.), subscripting suffices to save the distinction. For example, *torque* is the product of a force in, say, the *x*-direction $[F_x]$ and an arm length $[L_y]$ at a right angle. It may be dimensionally expressed as $[ML_xL_yT^{-2}]$. In contrast, *mechanical energy* – the product of a force and a length in the same direction – can be rendered as $[ML_x^2T^{-2}]$.

Provided the meaning of a scientific concept is treated as determined by the dimensions which constitute it (and their associated measurement procedures), a (natural) law can now be understood as the expression of a constraint on the distribution of points over the dimensions underlying a theory. Thus, in Newtonian mechanics it is predicted that all observed measurement points will lie on the hyper-surface spanned by $F = ma$ (Gärdenfors 2000). This is the basic empirical claim of the theory. Application-specific laws of mechanics,

8 For later changes of these domains, see section 5.

such as Hooke's law of the pendulum: $F = -\,\mathrm{k}x$,[9] add further constraints that increase the empirical content of the theory.

The conceptual spaces model treats all dimensions principally *on par*. The model leaves room, but does not demand to give epistemological privilege to fundamental over derived magnitudes. Nor do we insist on drawing Sneed's (1971) pragmatic distinction between T-theoretical terms (e.g., *mass* and *force* in Newtonian mechanics) and T-non-theoretical terms (e.g., *space* and *time*). However, in relying on measurement procedures to define the independence of domains, we follow Sneed's pragmatic stance: For a T-theoretical dimension, the value of an object on the dimension cannot be determined without applying the theory T itself. Finally, ontological questions which arise when assessing which dimensions to take as primitive ("Do forces exist?") are not answered by this model.

Conceptual spaces have been compared to the structuralist program in Gärdenfors and Zenker (2010). There we provide analogies to the structuralist's various kinds of models and constraints, arguing that the employment of set-theory paired with the use of theory cores is insufficient to account for theory change. In the following sections, the aim is to show that conceptual spaces fare better.

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[9] Here, x is the displacement of the spring's end from its equilibrium position, F the (material-dependent) restoring force, and k the force-(or spring-)constant.

4. Five Change Operations

When the framework of an empirical theory is modeled as a conceptual space, changes divide naturally into five types.[10] These operations suffice to trace change in a *finer grain* than distinguishing only between normal and revolutionary science. The changes are here presented in an order of increasing severity. What has commonly been referred to as a *scientific revolution* we primarily model by the last two changes, in particular the replacement of dimensions.

Like Kuhn's, our analysis of changes says little about the genesis of a successor framework. Rather, a general account is provided of *how* frameworks change. Greater scope and increased empirical adequacy may suffice in answering the why-question. Beyond such considerations, the incommensurability of frameworks, as it has been traditionally discussed, seems to pertain to ontological differences.

4.1 Addition and "deletion" of special laws

Examples of the perhaps most regular change of an empirical theory include special laws, such as Hooke's law of the spring (mentioned above), the law of the pendulum, $T = 2\pi \sqrt{L/g}$ (with T for period, L for the pendulum's length and g for gravitational acceleration), or Boyles' gas law, $p\,V = \mathrm{k}$ (with p for pressure, V for the gas's volume and k a constant).

[10] Gärdenfors and Zenker (2010) discuss only four types. In the present paper, "change of scale or metric as well as the salience of the dimensions" is divided into two types.

It may surprise that special laws are assigned a comparatively unimportant status in our classification. After all, predictions come about only in virtue of applying special laws and a *ceteris paribus* clauses (Zenker 2009). Moreover, as Kuhn (1961) stresses, the ever more accurate and precise determination of natural constants (e.g., Boltzman's: $[ML^2T^{-2}K^{-1}]$, [11] Hooke's: $[MT^{-2}]$ or of gravitational acceleration: $[LT^{-2}]$), on which predictions *depend*, accounts for a large part of normal science.

Nevertheless, once the dimensions which "go into a problem" are specified (if only hypothetically), formulating a special law may not be a surprising discovery. As Raleigh observed:

"It happens not infrequently that results in the form of 'laws' are put forward as novelties on the basis of elaborate experiments, which might have been predicted *a priori* after a few minutes consideration." (Rayleigh 1915; cited after Rosch 1998, 211)

So, given the conceptual framework is established, adding a new special law or improving the value of a natural constant is comparatively unimportant as a type of change.

In principle, special laws could be deleted. Thus, in the late 19th century, new exponents of r in Newton's law of gravitation $F = Mm/r^2$ (F being the net system force, M and m two masses, r their distance apart) were proposed regularly, e.g., $r = 2.00000016$ (see

[11] K is the dimensional symbol for temperature.

below). Yet, “deletion” is the wrong term. Rather, anomalous applications of special laws, e.g. Mercury’s orbit, are normally tolerated (Roseveare 1982). Possibly, a special law may “move” to another theory in which it is more successful. Thus, light had been supposed a mechanical phenomenon, but was successfully treated in electrodynamics. In brief, special laws are never literally deleted; rather, their scope of application is restricted. [12]

4.2 Change of scale or metric

The technical notion of *scale* goes back to Stevens (1946). He distinguished four levels (*nominal, ordinal, interval, ratio*) which received mathematical refinement by Krantz et al. (1971, 1989, 1990). The dimensions that constitute a domain are equipped with different metrics insofar as their scales “harbor” differentially specific information. With each scale-level, informational content increases as the next higher level shows fewer invariances.[13]

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[12] A “problematic” special law may also persist as part of a theory without any application being assigned to it. Dating to 1747, Clairaut’s law, $F = G\ Mm/r^2 + a/r^4$ (‘a’ being a constant) is a case in point (Chandler 1975). “Out of fashion” for about 100 years, it was reapplied (alas, without success) to Mercury’s orbit in the mid 19th century (Gähde 2002). In the structuralist’s terms, if T_n is the respective theory element to which the law is assigned, its set of applications $I\ (T_n)$ may “drop” to zero. Here, we follow structuralism’s pragmatic mode of identifying intended applications.

[13] As for the distinction between integral and separable dimensions, invariance classes are important also here.

For example, interval scales allow *linear* transformations, but ratio scales allow only *scalar* ones. Thus, the Kelvin scale entails that there is no temperature below zero degrees. In contrast, the point −273.15 degrees Celsius lacks a special status, as the Celsius scale is an interval scale.

The following quote from Hausdorf (1903) may serve in appreciating a change in metric as compared to more severe changes:

"We can guarantee that the dimension of [absolute] space is precisely three […]. Further, replacing the square by the exponent 2.00000016 (this has recently been proposed […] to explain the advance of the perihelion of Mercury) surely is an unfortunate idea. We cannot guarantee, however, that a measure of the curvature of space exactly equals zero […]; we know only that a very small […], positive or negative, number estimates that measure." (Hausdorf 1903, 2-3; cited after Czyz 1994, 251f.)

In this sense, changing a metric may qualify as a less severe change than conjecturing space to "have" four (or yet more) dimensions.[14]

[14] For the guarantee-part in the above quote, see Section 6 on Neo-Kantianism.

4.3 Change in importance of dimensions

Having mentioned the decreasing importance of *mass* above, we provide further examples. Until the 18th century, *color* as perceived by the eye remained important to analytical chemistry. Starting with Lavoisier's *oxygen chemistry*, the invention of instruments such as the polarimeter (later the photoelectrical detector) demoted the importance of perceptual color. In biology, pre-Linnaean botany focused on holistic dimensions of flowers, such as *size* and *color*, while Linnaeus' classification raised the importance of the *numbers of pistils* and *stamens* as salient classification features.[15]

The perhaps most important – and the ontologically most versatile – dimension of modern physics is *energy*. This notion was of hardly any importance to Newton's mechanics. Going back to Leibniz's *vis viva* as a term for kinetic energy, energy is revived in the 18th century by Young, becoming gradually more important in the development of physics. We return to this in the case study in Section 5.

4.4 Change in independence of dimensions

As defined above, a dimension is separable in a domain *C* if its invariance class does not involve dimensions from *D*. Traditionally, *mass* was an independent dimension. After Thomson's late 19th century observation that heavily charged particles apparently gain in

[15] Anders, Barker, and Chen (2006) provide several further examples of taxonomic change in biology and physics. Also see Zenker (2010).

mass, the mass dimension gradually loses its independence. In 20th century physics, mass and energy are effectively treated as integral dimensions.[16]

The paradigmatic 20th century example is the transition from Newton's 'space and time' to Minkowski's 'space-time' (ascribed to Einstein, prepared by Poincaré). In Newton's *absolute* 3-D (Euclidian) *space* and 1-D *absolute time*, there is no interaction in the respective measurements. With special relativity, spatial and temporal coordinates (*x*, *y*, *z*, *t*) are integrated. See the case study in the following section.

Another example is *heat*, which traditionally was framed as a substance – i.e., a fluid passing from warmer to colder bodies (e.g., *caloric*). In thermodynamics, this quantity effectively lost the independent status it has been assigned earlier when Boltzman "functionalized" temperature as *change in mean kinetic energy* (Chang 2004).

4.5 Addition and deletion of dimension

Staying with thermodynamics, in 1850, Clausius searched for the conserved quantity in heat change processes. Conjecturing *energia* to be some combination of *heat* and *work*, he introduced *energy* as a new dimension, integral with heat and work. In fact, already the 18th century Joseph Black distinguished the quantity of heat (*latent heat*) from its *intensity*, only the latter being measured by temperature.

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16 For a reservation concerning this statement, see section 5.

The 19th century saw attempts at reducing gravitational to electromagnetic forces (Roseveare 1982, Zenker 2009). Even today the term *force* is still used in many contexts. Alas, the three force dimensions of Newtonian physics [F^3] have ceased to be a proper part of a theory of relativity. And the early 20th century witnessed (mostly ignored) claims that the *relativistic mass* of a moving body is ill defined.[17]

Finally, consider the *ether* in its various forms (luminferous, electro-magnetic, etc.). The ether may be reconstituted as a 3-D vector space. It had been introduced – *qua* analogy with air and sound – as the medium carrying light. Following Michelson and Morley's null result and Einstein critically rendering it superfluous, the space [E^3] was effectively deleted.

The most severe change occurs when a dimension is added to a theoretical framework. The perhaps simplest example is the introduction of Newton's *mass* and *gravitational force* to replace Gallilean *weight*. In modern terms, an object's weight is analyzed as its mass under the influence of a gravitational field. It is interesting to note that, at least once, mass had been a candidate for being deleted as an independent dimension. Priestley resisted the Newtonian separation of matter and force. Instead, he proposed "to reduce matter entirely to the forces of attraction and repulsion" (McMullin 2002, p. 33).

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[17] "In 1948 Einstein privately cautioned in a letter that '[i]t is not proper to speak of the mass $M = m(1 - v2/c2)–1/2$ of a moving body, because no clear definition can be given for *M*. It is better to restrict oneself to the 'rest mass,' *m*. Besides, one may of course use the expressions for momentum and energy when referring to the inertial behavior of rapidly moving bodies.'" (Hecht 2009, 340).

Let this suffice to exemplify a historically regular process one might call "dimensional change". We next illustrate the five change operations in a case study, tracing the transition from Newtonian mechanics to special relativity.

5. Case Study

In this section we want to show that the transition from Newtonian mechanics to Einstein's special relativity is not as revolutionary as Kuhn and others have claimed. Within Newtonian mechanics a number of changes occurred that prepared the ground for special relativity. We will model these by using the five types of changes described in the previous section. Our aim is not to give a historical account, but rather to highlight some of the conceptual changes that occurred in this transition. Furthermore, we do not enter the discussion of the ontological status of the various dimensions.

5.1 Newtonian Mechanics

The quality dimensions of the conceptual space underlying the original Newtonian mechanics are ordinary space s (isomorphic to R^3), time t (isomorphic to R), mass m (isomorphic to R+), and force F (isomorphic to R^3). All spaces are Euclidean.

In the original version, Newton considered absolute space and time, which means the domains are assumed to have fixed origos. The theory is later reformulated without assumptions of absolute space and time. (This is a change of the second kind.) Instead it is described in terms of a relational space, *s*, invariant under linear (Galilean) transformations, and a relational time, *t*, also invariant under linear transformations. These invariances imply that *space*, *time* and *mass* are separable domains according to the criterion presented in Section 3.

Importantly, the total mass of the objects in a system is assumed to be constant over all applications of the theory. One of the new features of Newton's theory is that he introduces a distinction between *mass* and *weight*.[18] The dimensions of *velocity* $[LT^{-1}]$, $v = ds/dt$, *acceleration* $[LT^{-2}]$, $a = ds^2/dt^2$, *momentum* $[MLT^{-1}]$, $p = mv$, *kinetic energy*, $e_k = ½\, mv^2$, *potential energy*, $e_p = mgh$ or *work*, $w = fs$ (all three of the non-discriminate dimensional form $[ML^2T^{-2}]$), can be introduced as defined magnitudes in Newtonian mechanics.

The second law $F = ma$ $[MLT^{-2}]$ introduces a constraint which connects all dimensions of the conceptual space. It predicts that all observations of particle movements will lie on the hyper-surface defined by the second law. From this law, Newton is able to derive a number of fundamental mechanical laws, amongst others Galileo's and Kepler's laws, as well as the principle of the invariance of momentum. (Executing Galileo's vision, Newton thus unifies terrestrial and celestial mechanics.)

In the discussions of Newtonian mechanics through the centuries, one finds different opinions about the status of the domain of *force*. It can be taken as a primitive 3D space that is

18 In Newton's Principia, *mass* is officially defined in terms of density, which seems to play no further role in his theory.

separable from the remaining dimensions. In that case, Newton's $F = ma$ becomes a law introducing empirical constraints on observations. On the other hand, it can be seen as a defined magnitude, in which case the second law becomes the very definition.[19] On this interpretation, Newtonian mechanics will build on five dimensions: [M], [L^3], [T] (Kyburg 1984, 175ff). This choice can be described as a matter of the importance (salience) of the domains (a change of type 3).

The history of mechanics harbors proponents of both positions (Jammer 1957). Many wish to eliminate force as an independent magnitude, in particular Ernst Mach, but also proponents of electro-dynamic frameworks preceding him, such as Zöllner and Ritz (Zenker 2009, 50-54). *Hypotheses non fingo*, Newton seems to have viewed $F = ma$ as a law in which the fundamental status of the force domain is retained. As we have mentioned, Priestley, one of the last supporters of *phlogiston*, is an exception. As late as 1804, he denied it had negative weight. Following Boscovich, he sought to eliminate *mass* (or *matter*) – which "can be nothing else than the enumeration of its properties" (Schofield 1964, 293) –, but retain *force*. Boscovich's kinematics relied on distance and motion, the force of gravity being a derived quantity.

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[19] Gravitational forces are not technically necessary for Newtonian Mechanics to describe the empirical world.

5.2 Enter Energy

Leibniz had proposed the principle of invariance of kinetic energy (which he calls *force*). He argues that this is more fundamental than the invariance of momentum. In our classification of changes, this is an addition of a new law that increases the empirical content of the theory (change of type 1). Leibniz' principle has since been incorporated into classical mechanics (Lindsay 1971). Perhaps the first step in a conceptual change leading to the greater importance of energy or *vis viva* (living *force*), this invariance principle proves helpful as other types of energy are added (e.g., heat, electro-magnetic). This, however, does not strictly speaking extend Newtonian mechanics, which only handles potential and kinetic energies that are "gravitational."

Later mathematical reformulations of classical mechanics by Lagrange and Hamilton elevate energy. The Lagrangian of a system is its kinetic energy minus its potential energy. Furthermore, the value of the Hamiltonian is the total energy of the system being described. For a closed system, it is the sum of the kinetic and potential energy. In both cases, the role of forces is demoted. It is therefore reasonable to claim that the fundamental conceptual domains of Hamiltonian classical mechanics are *space*, *time*, *mass* and *energy*. This constitutes a change in the salience of dimensions (*force* decreases in importance, *energy* increases) (type 3). However the Newtonian and Hamiltonian versions of classical mechanics are formally intertranslatable and empirically equivalent. This once more points to the contentious issue being one of ontology – here the status of *forces*.

A step towards making energy more central and independent of Newtonian mechanics is Joule's (1849) determination of the mechanical equivalent of "heat" *via* the temperature-change in a liquid (Fig. 1).[20] The temperature change was effected by paddles rotating through the liquid, the rate of rotation being determined by the gravitational pull on a mass strung to the paddles. Measuring a vertical mass-displacement by a ruler is *independent* of measuring a temperature-difference in a liquid by a mercury thermometer, even when using the same ruler to determine the extent to which mercury expands. Hence, a conserved quantity, *energy*, becomes expressible that connects amounts of heat and work.

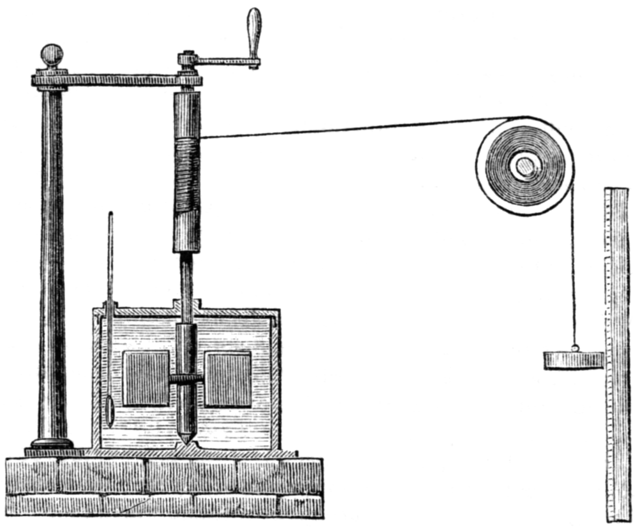

Fig. 1: Joule's apparatus for measuring the mechanical equivalent of heat[21]

[20] Sibum (1995, 74) holds that Joule's "exceptional experimental practice was based on the transformation of different, apparently unrelated traditions" and "thermo*metrical* skills which were rare in the early Victorian physics community" (*italics added*).

[21] Image: *Harper's New Monthly Magazine*, No. 231, August, 1869 (*public domain*).

In thermodynamics, *heat* (originally a fundamental dimension) is reduced to mean *kinetic* (motion) *energy* plus *potential* (bonding) *energy* of molecules, the latter accounting for aggregate-state transitions. Thus, the dimension of *heat* becomes derived; it can no longer count as separable within mechanics (change of type 4). The dimension of *temperature* thereby changes from an interval scale to a ratio scale (zero kinetic energy being the endpoint) (change of type 2).

Leibniz' principle of invariance of kinetic energy becomes the first law of thermodynamics. As regards the dimension of heat it can be asked whether "heat = mean kinetic energy" is a law or a definition. The situation is very similar to the interpretation of $F = ma$. Without discussing this issue any further, thermodynamics further contributed to the increasing importance of the energy dimension (see Clark 2002).

5.3 Electromagnetism

In *electrodynamics*, a new fundamental dimension, *electric current*, [I], and a derived one, *electric charge*, [IT], are introduced. Maxwell's equations express the connections between current, space and time. Also in this theory, conservation of energy is assumed to hold.

Thomson's discovery that the mass of heavily charged particles increases violates the fundamental assumption of Newtonian mechanics that mass is constant. Now energy is no longer seen as a defined entity as in Newtonian mechanics. With developments in thermodynamics (above), energy becomes an *independent* dimension of increasing importance (change of type 4). The next change is that first Poincare, then Einstein argues that mass and energy should no longer be viewed as separable dimensions (another change of type 4). By the principle of the *inertia of energy*, to each energy, *E*, there corresponds a (rest-)mass $M = E / c^2$, where "*E* need not be the total energy of a system, as was first hypothesized, but […] a mass (or momentum) may be associated with each individual energy (or energy current)" (Hickman 1984, 542). Eventually, each field of force is assigned its potential energy.

A consequence of the development of electromagnetic theory is that *fields* become important representational formats (McMullin 2002), e.g., the electric, [E^3], or the magnetic field, [B^3]. However, fields can be seen as a special type of conceptual spaces. For example, a scalar value from a dimension D (e.g., temperature) being assigned to a point in a space C (e.g., 3D Euclidean space) can be seen as a 4D conceptual space, $C \times D$, with the addition of a function from C to D that specifies the values. Similarly, a vector field, *F*, assigning a 3D vector (e.g., *force*) to a point in a space C (e.g., 3D Euclidean space) can be seen as a 6D conceptual space, $C \times F$, with the addition of a function from C to F specifying the vectors. A similar story can be told for tensor fields. In this way, forces become "hidden" in vector fields. In brief, the use of fields simplifies certain representations, but they are still compatible with the description of theories in terms of conceptual spaces.

The Lorentz(-Fitzgerald) transformation were first obtained by Lorentz when he sought to find the transformations which left Maxwell's electro-magnetic equations unchanged in form. The transformations were interpreted as contractions of objects in their direction of motion.[22] Space and time are still considered separable and Newtonian. However, no physical mechanism could be discovered to account for the contraction of objects. Poincaré's mathematical insight and Einstein's bold move led the transformations to express an intrinsic property of space and time, or rather space-time. Poincaré favored upholding the aether, Einstein deleted it. His eventual mathematical device, the energy-stress tensor, no longer reminded of anything substance ontological, but is properly called functional.

5.4 Special Relativity

Einstein starts from two fundamental postulates: (1) The speed of light is constant in all inertial frames of reference; (2) all laws of physics (here: electrodynamics) are the same in every inertial frame. The first postulate is the fundamental new constraint of his relativity theory. From these postulates, he is able to show the invariance with respect to the Lorentz transformations.

Accepting these transformations means that space and time are no longer separable domains, but form an integrated four-dimensional space-time. Since the geometry of space-time is different from that of space and time in classical mechanics (Minkowskian *vs*.

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[22] "Tweaking the scales" would have been equally possible: "It is only the relation of the magnitude to the instrument that we measure, and if this relation is altered, we have no means of knowing whether it is the magnitude or the instrument that has changed" Poincaré (1897, 97).

Euclidian metric), this involves a change of type 2. Furthermore, unlike Newtonian theory, the passing of time can no longer be independent of the state of motion of an observer. Thus, Einstein's postulates entail a change in the separability of dimensions (change of type 4). However, *mass-energy* is still separable from space-time.

Einstein then adds a third postulate concerning the conservation of momentum: (3) The total momentum of a system is preserved in all inertial frames of reference. In this context, momentum becomes $p = mv / \sqrt{}(1 - (v^2 / c^2))$. In contrast to Newton's theory, this principle entails the conservation of total energy of a system. Provided energy, "massless objects" now travel at the speed of light, *c*, while accelerating a mass to above *c* requires infinite energy. Energy and mass become exchangeable and no longer count as separable dimensions (type 4).

Furthermore, the relativistic mass of an object is dependent on the inertial frame. What remains constant is the rest mass of an object which does not move in relation to a given frame. However, when objects in a system fall into parts, e.g., in fission of radioactive particles, the sum of the objects' resting masses need not be constant. This should be contrasted with mass as an *invariable* quantity in Newtonian mechanics.

The dimensions of special relativity thus are *space-time* $[L^3T]$, *energy* [E] and *mass* [M]. The relativistic momentum and electromagnetic field-forces are derived dimensions, which accords with the development of mechanics after Newton. Later, with General Relativity, gravitation enters, and mechanics is unified with electro-magnetism.

5.5 Summary

The key question in relation to Kuhn's incommensurability claim is how the "meaning" of *mass* changes from Newtonian to Einsteinian theory. What remains constant over time is that mass is one-dimensional and measured on a ratio scale. The physical interpretation of rest mass seems to remain fairly constant as well. What changes is the separability of mass from energy and its frame dependence, that is, that the sum of (resting) masses is not constant for a system. Further, *energy* gains in importance, while *force* is demoted. This should be more informative than learning of a radical shift in the meaning of mass. Meaning resides in the conceptual structures and their measurement procedures, not in the symbolic laws that are formulated to express connections between the dimensions.

The gradual transition of conceptual spaces from Newton to Einstein can be illuminated by our analysis of the types of changes to the framework. The upshot is that this transition is much less *revolutionary* than Kuhn claims.

6. To what Extent are Conceptual Spaces Neo-Kantian

It can be said that our use of conceptual spaces is a form of Neo-Kantianism, since the dimensions express the "Anschaungsformen" for what is investigate within the theory. In this section we will compare it to other modern forms of Neo-Kantianism.

Above, we pointed out that within the conceptual spaces approach, no principled distinction is drawn between *fundamental* and *derived* dimensions. Rather, we side with

Einstein:

“[O]ne must not (…) speak of the ‘ideality of space.’ ‘Ideality’ pertains to all concepts, those referring to space and time no less and no more than all others. Only a complete scientific conceptual system comes to be univocally coordinated with sensory experience. On my view, Kant has influenced the development of our thinking in an unfavorable way, in that he has ascribed a special status to spatio-temporal concepts and their relations in contrast to other concepts.” (Einstein 1924, 1690f.).

We find a similar influence in contemporary Neo-Kantianism. Notably, Michael Friedman (2008, 239) accepts incommensurability between historically successive frameworks on the basis of meaning-divergence.[23] Continuity, on the other hand, is found in a “continuously converging progression of abstract mathematical structures framing, and making possible, all of our empirical knowledge” (241). The important qualification is that “the convergence in question occurs entirely *within* the series of historically developed mathematical structures” (242), rather than with respect to objective reality.[24]

[23] Kuhn was “a Kantian with moveable categories” (Hoyningen-Huene 1993; Kuhn 2000, 264). On Kant’s a priori grounding of action-at-a-distance, see McMullin (2002, 29ff).

[24] According to Friedman, Kuhn may be placed within Cassirer’s *genetic conception*. It sees scientific knowledge “progress from naively realistic ‘substantialistic’ conceptions, focusing on underlying substances, causes, and mechanisms subsisting behind the

Friedman's view might appear congenial to ours. After all, analytic knowledge is viewed as a matter of the particular conceptual space ("analytic-in-S").[25] We part company by making less of *revolutions*. Thus, Friedman's tri-partition into (i) empirical laws, (ii) constitutively a priori principles making them possible, and (iii) philosophical meta-paradigms which "provide a basis for mutual communication […] between otherwise incommensurable

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observable phenomena, to increasingly abstract purely 'functional' conceptions" (2008, 244). *Structure*, particularly the notion of *paradigm shift* and (to a lesser extent) the subsequent replacement for paradigm – the *structured lexicon* – draws support from diverging ontologies. Therefore, Friedman argues, Kuhn's position also(!) reflects the Meyersonian substantialistic view, which is directly opposed to Cassirer's. Unsurprising, then, that his readership remained divided between support and opposition.

Rudolf Carnap, series editor for *Structure*, expressed sympathy for Kuhn's ideas (Reisch 1991). After all, Carnap's philosophy of linguistic frameworks – not known to Kuhn in great detail – "is wholly predicated on the idea that logical or analytic principles, just as much as empirical or synthetic principles, can be revised in the progress of empirical science" (Friedman 2002, 176). On Reichenbach's (1920/1956) "relativized and dynamical conception of the a priori" (175), Friedman holds that Euclidian geometry and the instantaneous propagation of light-signals count as constitutively *a priori* elements of Newtonian physics. The elements provide the framework making empirical knowledge (as expressed, e.g., in the law of gravitation) possible. Thus, 'a priori' denotes 'necessary' in the sense of 'indispensable for empirical knowledge', and no longer carries the meaning 'unrevisable'.

25 For example, it is analytic in the space of Newtonian Mechanics that all masses are non-zero and that no change in acceleration arises without a force. In analogy with Kuhn, *analytic-in-S* pertains to whatever cannot be questioned in a paradigm.

(and therefore non-intertranslatable) scientific paradigms" (Friedman 2002, 189) becomes suspect (also see Howard 2010). What use is there for "meta-paradigms," lest revolutions were framework-reorganizations that necessarily lead to communication breakdown?

The third level can be easily dispensed with, once a conceptual framework is understood as a conceptual space.[26] We claim that, beyond the five change operations, no other tools are needed to reconstruct the conceptual part of scientific change. Even severe changes thus turn out to be *less radical* than Kuhn and his interpreters might seem to believe. After all, if applications (e.g., Mercury's orbit) are shared between frameworks, it is possible to compare the new and the old space *vis á vis* this application. Consequently, the respective conceptual spaces with their different properties can be defined via the change operations.[27]

Such transitions are in no good sense instances of a rationality-defying shift in knowledge. Mathematical advances (e.g., from vector to tensor calculus in general relativity) do not change the underlying conceptual space, either. As we saw, the *conceptual* change in the transition from Newtonian mechanics to special relativity consists in integrating 3D-space and 1D-time into 4D space-time and making *energy* and *mass* convertible.

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[26] Pragmatic factors that pertain to applying a theory successfully do not fall under conceptual knowledge. Likewise, alternative or near-alternative formulations of empirical theories are easily mistaken for an alternative conceptual space which they are not.

[27] If more is needed for communication, it might no longer relate to conceptual representation but, perhaps, to human imperfection.

Finally, our proposal to reconstruct scientific change in conceptual spaces is not "whiggish" either. The development of scientific knowledge does clearly not consist in *adding* facts. Rather, integration of prior theory is the better term. Conceptual spaces handle such complexity up to *n* dimensions. Alas, the model will not tell us *how* to develop an empirical theory (though it might suggest as much).

7. Conclusion

Our starting point has been that describing changes of scientific theories in terms of symbolic laws is not the appropriate strategy. On the other hand, replacing it with the dichotomy between normal and revolutionary science obliterates many changes within what Kuhn describes as normal science that prepare the conceptual ground for a more radical shift. Conceptual spaces as a reconstructive framework for empirical theories allow a more realistic description of how scientists work than what is offered by these two accounts. The model also provides a more fine-grained analysis of scientific change than Kuhn's dichotomy between *normal science* and *revolution*. On this view, empirical theories are understood as collections of structured dimensions. Scientific change then denotes their systematic modification. In principle such change can be fully reconstructed.

A reconstruction in conceptual spaces serves to get clear on the dimensions which "go into a problem." We adopted this idea from dimensional analysis and extended it to framework revisions. We can therefore (rather easily) reject the interpretation of a paradigm-shift as a non-rationally reconstructable change-episode, the rationalization of which *must* cite extra-scientific factors (power, fashion, interests, etc.). Once the tool is rich enough to describe the basic changes, rationality no longer displays gaps. Here, the basic move is to identify an empirical theory not via a core, but by using dimensions in a more direct way.

In the construction arrived at through a sequence of dimensional changes, we showed by historical example that scientific theory change is a regular process. On our proposal, any possible change should "leave a trace" by exemplifying at least one of the five types of changes. Only the latter two (change in separability and addition/deletion of dimensions) seem to indicate a *radical* change. We invite historians and philosophers of science to apply the dimensional analysis and our categorization of changes to other cases to test their viability.

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