Relativity concept in Mach with the use of Dynamic Frames

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Abstract

In this article we will analyze the concept of relativity in the Machian work using *dynamic frames* provided by the Cognitive Science and now used successfully in Philosophy of Science. We will emphasize that the concept of relative can be divided into three types and with these new tools we will address the Machian interpretation of the Newton's bucket experiment.

1 Introduction

The concept of relativity has been widely studied in the history of ideas and especially in reference to the study of motion. Following the scientific revolution in 600/700 it was formed a world view based on Newtonian mechanics, where the notion of relative was related only to the concepts of position and speed, while acceleration and force (excluding the inertial forces) was something absolute as we can deduce from the arguments of the English scientist.

Already at the birth of Newtonian mechanics there were the supporters of a more relational view of movement such as Berkely[18] with his comments on the concept of motion[1], the philosophical criticism of Leibniz[25][2] and some observations present in the important essay on Celestial Mechanics of Lagrange [22, 21] [3], that suggested a rethinking of mechanics on other bases.

This historic shift occurred in the second half of the 800 when physics began to move away from the mechanicistic view of the world, dealing with new conceptual tools phenomena hitherto unknown (such as electromagnetism and heat). Newtonian mechanics was analyzed and criticized - as is known - by

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[1]Consider for example the essay *De Motu* where he proposes a criticism to the concepts of absolute space and time at the base of newtonian mechanics

[2]Consider especially the spatial concept of Leibniz where only the relative distances among objects have an ontological value.

[3]In 1772 Lagrange had determined the equations that govern the motion of three celestial bodies in interaction with each other, showing that they depend not only on the mutual distances and on the first and second derivatives with respect to time, as one would expect from newtonian physics, but also on the third derivative.

the Austrian philosopher and scientist E.Mach that put part of the conceptual foundations of the relativistic revolution of the twentieth century.

Mach's influence on young scientists of the second half of 800 is a subject discussed in detail in the last 50/60 years, so there is a large literature on it [3, 34, 20] and we also treated it in a previous article [16] some years ago. Greater attention has been placed on the influence that the thought of Mach had on Eintein in the formulation of special and general relativity, and especially of what is normally called Mach's Principle [17, 33, 34]. In an article of 1912 [11], which deals with the influence of a spherical shell of dust on a massive body in it, and as part of a static theory of gravitation, Einstein proposes that principle - although the actual name will be recognized only in 1918 [12] - stating that the entire inertia of a body is somehow determined by the presence of other masses of the universe and cites Mach as his source of inspiration. The fact that Mach has actually made such a principle, even just in a heuristic form, is a problem that has been addressed many times by historians of science [4] , everyone coming to the conclusion that this claim is never present in the machian writings although several sentences seem to imply it.

In this regard it is simple to mention one of the most controversial sentence of the "*Mechanic in its logical and historical development*" [27] involving the famous experiment of Newton bucket:

> "*Newton's experiment with the rotating vessel of water simply inform us, that the relative rotation of water with respect to the sides of the vessel produce no noticeable centrifugal force, but that such forces are produced by its relative rotation with respect to the mass of the earth and other celestial bodies*" (1)

where it can easily observe how the centrifugal forces inside the bucket are '*produced*' by the rotation of the water respect to the other masses of the universe, as if the latter caused the former. From these types of sentences there were a flowering of researches, both theoretical and empirical [15, 14, 13, 19] which had as its starting point the deep analysis of the Austrian philosopher. The solution of the problem generally is achieved by analyzing the controversial sentences in the light of Machian philosophy: in fact the latter is based on the concept of the relationship among the elements of the universe and when Mach addresses a physical problem, he has always tried to describe the phenomenon highlighting the various elements that constitute it and the relationships among them. For Mach the scientist has to build a model of what is studying, emphasizing some aspects and leaving many others in the background [31].

To effectively represent the concept of relativity proposed by Mach and to further clarify the historical background that we have briefly introduced, in the following we will use the "*Dynamic Frames*" made available by the Cognitive Science. They are a valuable tool to describe and formalize what a concept is, and we will use their richness to describe a cognitive structure (as may be an acquired knowledge), omitting their use in understanding the cognitive processes

[4] see previous documentation

of the human being - and the scientist in particular - that is the basis of their success in Philosophy of Science [32].

2 Dynamic Frames

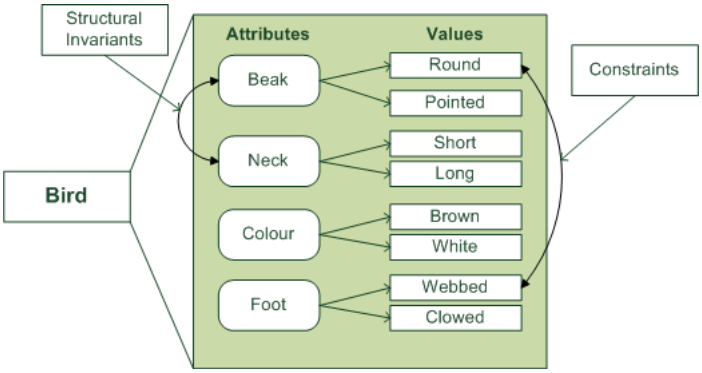
The classic definition of what is a concept has its roots in Aristotelian logic and can be summed up in three simple assumptions: (1) The representation of a concept is an outline of a whole class of instances that fall below it; (2) The features that represent a concept are necessary and sufficient conditions for its definition; (3) The characteristics are incorporated into each other, so for example if a concept C is a subset of a concept Y, all the characteristics of Y are included in C [39, 24]. This definition considers concepts as 'features list' and a classic example is the definition of man as [bipedal, rational animal]. You can easily seen that this definition is in crisis with a simple counterexample: if you consider a Paralympic athlete he does not belong to the definition just given, but common sense leads us to contradict this conclusion. To take a further example, if we think about the concept of the square as the set [4 equal sides, 4 interior angles of 90°], it is no longer valid if we consider a curved surface; in this case the inner corners can be different from 90°. The classical theory of concepts has been questioned by Wittgenstein in the *Philosophical Investigation* [38], where the Austrian philosopher pointed out that a concept can not be defined by a set of necessary and sufficient conditions because even the most common concepts, as the play, do not possess unique common properties. Rather by the idea of '*Family Resemblance*' objects, that fall under the same concept, share properties which partly overlap, but have totally nothing in common; note also that these properties are defined by language practice. The idea of Family Resemblance was studied in philosophy since the 60s, but only the work of Kuhn [23] fully availed of its effectiveness. Yet in the field of cognitive psychology the idea proposed by Wittgenstein had a great success thanks to the work of E.Rosch in 70s, where it was discovered that human concepts had a '*graded structure*' [35, 36] . As a result of numerous experiments - especially on natural objects - it was possible to point out that humans determine at what level one instance of a concept is a good or bad example of the concept. To do also in this case a simple example, the concept of bird is represented in a better way from a robin rather than by a penguin - although the latter falls into the same category.

Rosch studies led to the birth of a number of tools in order to represent concepts; among these we will use the '*Dynamic Frames*'[5]. In the early 70s the notion of frames was used by numerous scientists in Artificial Inteligente to simulate human behavior in everyday activities and around the 80s the tool was deepened and changed from Barsalou in his studies on categorization, autobiographical memories and finally on the effect of context in the representation of concepts[6]. Barsalou also changed the name from Frames to *Dynamic Frames*.

[5]Frames were introduced for the first time in the studies on memory of the British psychologist F. Bartlett[10]

[6]For studies on categorization see[4, 7], for autobiographical memories consider [6] and

A Dynamic Frame is a diagram (and the feature list of the beginning of the paragraph is a simple example of it) with a series of attributes which belong to the concept in question and each possible value they possess. To do the classic example present in the work of Andersen, Barker and Chen [1], if we consider the concept of a bird we know that it is constituted by a set of attributes/values shown in the following figure.



where attributes are on the left column and the values on the right. The diagram is a partial representation of the concept, because there will be numerous attributes that have not been indicated: in essence does not exist a complete and exhaustive representation of the attributes of a concept, but it is certain that those shown in the figure are sufficient to fix a category. A key aspect to consider is that values have a trigger point depending on the example that is considered: so if we analyze a sparrow as a representative of the concept of bird it will have the pointed beak (Beak = POINTED), the neck short (Neack = SHORT), the colour brown (Colour = BROWN) and feet clowed (Foot = CLOWED); everything can be summarized in the following list:

Sparrow = [beak-pointed, neck-short, colour-brown, foot-clowed]

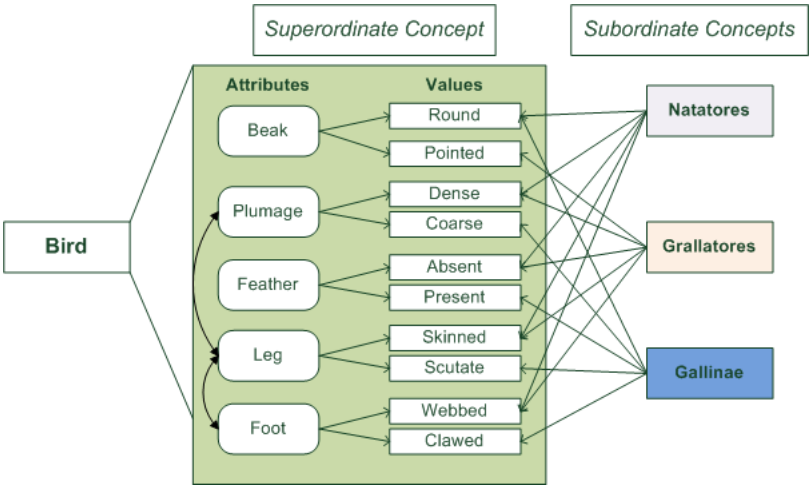
There are other relationships between the nodes that make up a concept: for example there are connections among the attributes, which are called '*structural invariants*'. In our case everything that has a beak also has a neck, but it is not sure that those who have feet also possess a beak. These structural relationships - which in our example are imposed by nature - are shown in the frame through curves linking attributes. Equally interesting are the '*constraints*' that exist among the values. So for example in the water birds that possess webbed feet (foot = WEBBED) you always have the beak is rounded (beak = ROUND): these constraints are represented in the figure with curved lines that connect the values involved.

An interesting aspect to consider is that the frames are recursive in the sense that each node can be a concept that is in turn represented by a Frame. In any case, the recursive nature of the frames does not lead to a basic level with

finally for the concept representation you can see [5]

respect to which it can not be further 'step down'[7] Finally consider that there is no single - unique - frame relative to a specific concept, but there are equally sustainable ones: the goodness of a frame is evaluated - at the end - according to its empirical adequacy.

It is interesting to note that any frame can be represented through a multidimensional space; it is sufficient that each attribute with its possible values is represented by a spatial dimension. For example the concept of [BIRD] is represented by a 4-dimensional space where each dimension - one for each attribute - assumes bynary values[8] The advantage of see a frame under the aspect of a multidimensional space can lead us immediately to introduce a metric to identify the affinities between the various concepts.
So, if we consider the Sundevall taxonomy for the birds classification [1] we will have the following dynamic frame:



The same informations can be grouped in the following lists:

Natatores = [beak-round, plumage-dense, feather-absent, leg-skinned, foot-webbed]

Grallatores = [beak-pointed, plumage-dense, feather-absent, leg-skinned, foot-webbed]

Gallinae = [beak-pointed, plumage-coarse, feather-present, leg-scutate, foot-clawed][9]

Both with the dynamic frame that with lists it is easy to see how the subconcepts Natatores and Grallatores are identical unless the value assumed by the attribute [beak], whereas the subconcept gallinae is the most different (*distant*) from other subconcepts.

[7] In essence there isn't an atomistic view.

[8] The fact that the attributes take only binary values, and consequently the relative dimension has only two possible values, should not worry because it is tied to the example considered .

[9] The **Natatores** category contains swans, geeses, ducks...; the category **Grallatores** includes herons, screamers, storks... and finally in the **Gallinae** category we have chickens, turkeys, quails..

So if we impose that each attribute can take the values $\{1, -1\}$in correspondence of the possible values assumed, we will have the three subconcepts are represented by the following vectors:

$$Natatores \Rightarrow \overrightarrow{Nat} = (1, 1, 1, 1, 1)$$

$$Grallatores \Rightarrow \overrightarrow{Gral} = (-1, 1, 1, 1, 1)$$

$$Gallinae \Rightarrow \overrightarrow{Gall} = (-1, -1, -1, -1, -1)$$

and if we consider the euclidean metric, we have that the distance (*similarity*) among the three concepts are the following:

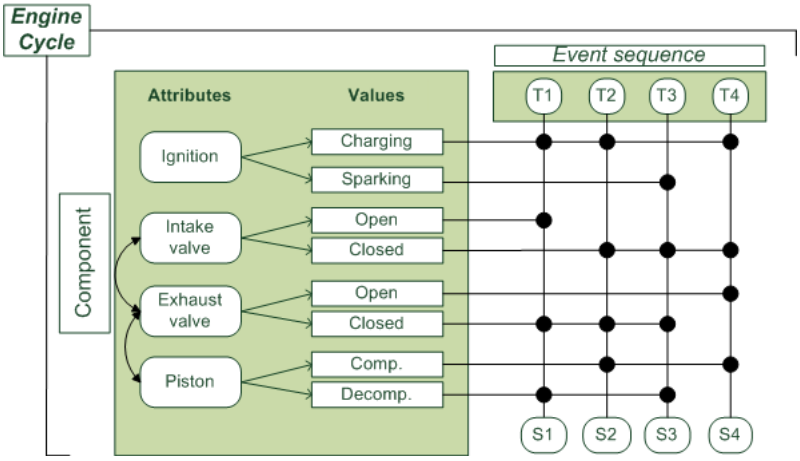
$$d\left(\overrightarrow{Nat}, \overrightarrow{Gral}\right) = 2.0$$

$$d\left(\overrightarrow{Nat}, \overrightarrow{Gall}\right) = 4.47$$

$$d\left(\overrightarrow{Gral}, \overrightarrow{Gall}\right) = 4.0$$

where we can easily see that Gallinae concept is the most distant (therefore the most *dissimilar*) than the other two subconcepts, while the more similar are the concepts of Natatores and Grallatores.

So far we have used and discussed '*static*' concepts, which are part of a taxonomy and take the name of '*object concept*', but cognitive scientists have also introduced '*dynamic*' concpts that are called '*event concept* ' and that will be useful in the analysis of the Newton's bucket experiment. These concepts can be represented by the structure of dynamic frames.
In studies carried out by Barsalou and Sewell [9] it has shown that whenever an 'event concept' is used, stored ... this is processed in a different way than what is observed for the 'object concept'. More specifically, it seems that the time relationship inherent in the 'event concept' are not represented by attributes, but organized according to a timeline / chronology; if this is so - and in this regard cognitive research has not given a definitive answer - you need to change the dynamic frames to represent these cases. Following the proposals by Barsalou [8] we could use a dynamic frame whose attributes take on different values depending on the sequence of events that you consider. To exemplify what has been said, we report the paradigmatic example provided by the author himself and which involves the cycle of the combustion engine: the dynamic frame is the following



where on the left there is an '*object frame*' which shows the components of a combustion engine and the possible states that they may assume (*component frame*). At the top is present the sequence of events that characterizes the event concept - in our case a combustion engine of 4 stroke - and for each instant T_i are shown the values assumed by the attributes that belong to the *component frame*. Finally in the lower part are indicated the subconcpts (*stroke engine*) that are identified for each instant in time sequence [10].

We now possess the conceptual equipment needed to deal with the concept of relativity (relative) in Mach's work which we analyze in the next section.

[10] We will not enter here into further details about event concepts and refer to relevant literature.

3 The concept of relativity in Mach.

In this section we want to analyze the concept of relativity in the Machian work using Dynamic Frames presented above. This analysis will be useful to highlight some aspects of the Machian philosophy that will allow us to understand the correct meaning of his most controversial sentences that are the basis of our historical problem.

The idea of relativity - or rather of relational - is a key concept of the Machian philosophy, although the same Mach never expresses a precise position on this term, or at least not in the meanings that the concept took over after the Einsteinian revolution. Machian position - as we have presented in the introduction - has its roots in the relational critic to the definition of motion proposed by Newton and developed especially by Leibniz and Berkeley, but appears to be more organic and based on the scientific discoveries occurred during the 800 (thermodynamics and electrostatics especially). As a whole the thought of Mach is called *Neutral Monism* [2] and had a considerable influence both on W. James and B. Russell for not forgetting the inevitable foundation that represented for the philosophers of the Vienna Circle. The focal point of this philosophical approach is to assume that the experience (as a whole, therefore including the human and emotional sphere) consists of elements of the same type that have no specific characteristics, in fact Mach names them as '*elements*'. Perhaps the easiest way to introduce them is to bring back a step of "*Popular Scientific Lectures*" [30]:

> "*Let us look at the matter without bias. The word consists of colours, sounds, temperature, pressures, spaces, times and so forth, which now we shall not call sensations, nor phenomena, because in either term an arbitrary, one-sided theory is embodied, but simply elements. The fixing of the flux of these elements, whether mediately or immediately, is the real object of physical research.*"

Although elements of the experience do not have specific characteristics, they can be divided into three groups:

- ABC: elements of ordinary things; for example tables, chairs etc ... so elements that represent what usually is defined as the outside world.
- KLM: elements that constitute our body; for example the retina of the eye and the nervous apparatus.
- $\alpha\beta\gamma$: elements that form our mental representations; for example the feeling of happiness in doing an action, the mental representation of an object that is no longer present at the sight etc ..

The splitting we have proposed - presented by Mach in '*The Analysis of Sensations*' [29] - is purely conventional, but it allows a useful subdivision. That is a simple convention is evident from the following example: if we consider a burning candle in front of us, in order to represent fully the phenomenon

we should consider the elements that make up the candle (the wax which is composed of), the elements relating to the light emitted (elements ABC), the sensory surfaces of our body (elements KLM) that receive the effects of external objects (the retina of our eye to make a simple example), the nerves that transfer the information from the peripheral to the center system and finally the mental representation we make of it (elements $\alpha\beta\gamma$). The determination of the candle as an external object, so as object studied by physics, is only because we make a filter on the relationships among the various elements that make up the experience, focusing only on few ones and omitting many other; in our example we do not consider the relationships between ABC and KLM (and therefore $\alpha\beta\gamma$). This simple example shows also a fundamental aspect for our analysis: a single experience, even the most simple as can be the observation of a burning candle, is extremely complex and is formed by a set of elements in mutual relation. The fact that in the study of the outside world a number of relations is left out is allowed by the following consideration.

Taking a cue from thermodynamics - so the study of closed complex systems - Mach says that ultimately the relationship among the elements of the experience can be represented by the following equation:

$$f\left(ABC\,,\,KLM\,,\,\alpha\beta\gamma\right)=0 \tag{1}$$

where the various elements interact with one another and the sum of interactions has a null result (just as in the case of thermodynamics). If this is the overall representation of an experience, we can study only a part of it - the external world for example - noting that the effects of the internal elements, as well as the sensory apparatus, does not determine the occurrence of an experience. In this way, we reduce the experience to a function of type:

$$f\left(ABC\right)=0 \tag{2}$$

that is an equation analyzed by physics.

If we observe in more detail the two equations we find they indicate that when a group of elements achieves a variation, another tries to compensate it, in such a way the result is null. This aspect is critical to avoid the possibility of sudden changes on physical elements; when a process creates a difference, the fact happens just because another difference is decreased. At the end the equation indicates that physical changes should be subject to the principle of the *absence of perpetuum mobile*.

The approach described above is borrowed from the historical study of thermodynamics: in "*The Conservation of Energy*" [28] Mach offers some analogies that explain the relationship among the elements of the experience and that are the cornerstone of his later research; for example we report the following sentence:

> *"S. Carnot Found that whenever heat performs work, a certain quantity of heat goes from a higher temperature level to a lower one. He supposed in this that the quantity of heat remains constant. A*

simple analogy is this: if water [...] is to perform work, a certain quantity of it must flow from a higher to a lower level: the quantity of water remains constant during the process... Electricity can perform work when it flows from a body of higher potential to one of lower potential: the quantity of electricity remains constant. A body in motion can perform work if it transfers some of its vis viva to a body move more slowly. Vis Viva can perform work by passing from a higher velocity-level to a lower one; the vis viva then decreases".

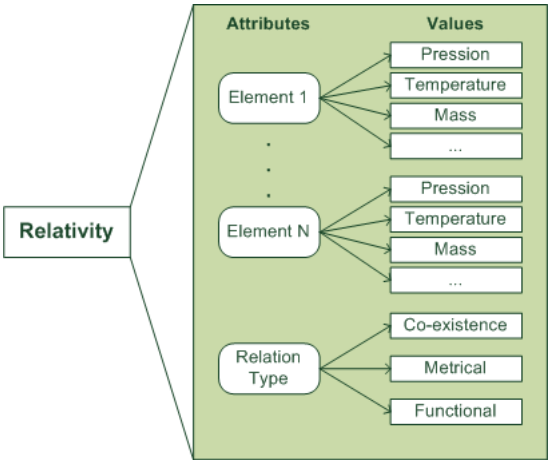
This phrase indicates clearly that in Machian thought is not significant that the amount of substance (heat, electricity, ...) is maintained, but that all energies can be represented as combinations of natural potentials that cause the variation in intensity of what they represent (the temperature in the case of heat). These natural potentials are the correct description of the '*datum*' provided by experience and not the eventual metaphysical constructions that often form the basis of our worldview. In this regard already in the article "*Bemerkungen uber die Entwicklung der Raumvorstellungen*" [26] of 1866 Mach heavily criticizes the mechanical concepts of space and time, assuming that the elementary experience of physics is given by the forces / pressures that are measured and so that the concepts of newtonian classical mechanics should be expressed through them. So for example in the above article Mach affirms the importance of the concept of force and its independence from the spatial relationships on which generally is defined, observing that "*... Now It seems to me that the fundamental law of force in nature need not contains the spatial relations of the pieces of matter, but must only been in dipendence between the states of the pieces of matter.*" The concept of force / pressure is to assume the role of temperature inside the building of thermodynamics, so a natural potential directly observable that describes the relationships among elements. Space and time, already in the article of 1866, are derived concepts: in fact when we say that a concept of physics is 'a function of time', we are saying that it depends "*on the position of the pendulum swaying pendulum position on h of the rotating earth ...* ", so on a secondary movement taken as a sample. If then we consider that the positions of the objects can be recognized only by their physical states, we are simply saying that "*all the states of the material universe depend upon one another.*" Therefore to Mach the concepts of space and time and their use in Newtonian mechanics indicate the mutual relationship among the elements of experience.

This last consideration is taken to extremes in Mechanics, where the same acceleration is interpreted as '*caused*' by the change of a natural potential. In fact if we consider bodies very distant from each other which move with constant direction and speed, compared to far fixed bodies, we note that they vary their distances in function of time. We can also say that very distant bodies mutate their mutual distances in such a way that they retain their proportionality. For example, if we consider the case of two distances r and ρ we will have the relationship $dr/d\rho = cost.$ Suppose now the masses in interaction, in such a way that at least an acceleration exist $d^2r/dt^2 = a$ and remember that time in the denominator can be expressed as the measurement of the distances between

the celestial bodies - as is already indicated in 1866. The acceleration thus assumes the form $d^2r/d\rho^2 = a$ and it represents a movement from the privilege state expressed by equation $d^2r/d\rho^2 = 0$. The acceleration is thus thought as a potential and brought back to the relative positions between the heavenly bodies.

An important aspect to consider when analyzing the concept of the relationship among the elements is that these relationships are part of the experience, are deducted from it and they have never posed by the scientist to organize the existing: what is provided by experience is a set of elements (colors, sounds ...) merged with their reciprocal relationships.

So far we have presented - in a more or less explicit form - the thought of Mach on what are the elements of the sensitivity and their mutual relations; now we try to systematize what has been presented in a dynamic frame that faithfully represent the concept of *relativity* in Mach. To do this we must first determine what are the '*attributes*' of the concept and later the possible values. Since the Machian philosophy involves the elements and relationships between them, the attributes of the concept of relativity will be the elements that are considered from time to time and the type of relationship that exists between them; all this can be represented by the following frame:



where the concept of relativity has a number of elements as 'attributes' that can assume each certain 'values'- such as Pression, Temperatures, Mass - and where is defined a type of relationship existing between the elements that, as we have previously presented, it may take three kinds of values: '**Co-existence**', '**Metrical**' and finally '**Functional**'.

The first relation, of coexistence, is to indicate that two elements are in the easier relationship that exists, the co-existence. When we look at an object and study its motion, generally we abstract the experience leaving on background the set of bodies with respect to which the body moves. The bodies left in the background - that represent the objective reference system , against which we evaluate the motion - are such because they co-exist with the body in motion,

even if they have no influence (at least apparently) on it. The relationship of coexistence between the various elements of the experience is a based relationship: any element we consider is always in a relationship of co-existence with some others. As stated by Mach in '*Knowledge and error*' [31] "*Even a body, [...], belongs to a complex and so to the world; nothing exist in isolation* ".

The second type of relation between the elements of the experience, what we have defined as '**metrical**', already involves some concepts of physics: so for example if we consider the table present in the kitchen and affirm that it is 2 meters long we are actually setting a 'metric relationship' between the table and the test sample (the meter). We can do a more complex example and resume the short analysis of the concept of time that we have done little above: even in this case when we say that an event occurs at an instant of time, we are actually establishing a metric relationship between the event considered and the rotational motion of the earth, or if we want the oscillatory motion of the pendulum. In a similar way also the concept of temperature is the expression of a metric relation; in fact, when we state that the temperature of a glass of water is equal to 15° we are actually comparing the state in which the water is found at that particular instant with respect to two other states: the melting point of ice and the point of boiling water.

More complex appears to be the third relation. It represents as a whole what generally goes under the name of physical law and that we have expressed with the formula (2): to make also in this case a simple example it is sufficient to consider the Boyle's Law ($PV = kT$) which relates the pressure, volume and temperature of a perfect gas. Under this type of relationship - that indicates a relation of physical interaction - are covered not only the 'quantitative' relationships between the elements of experience as may be the law just described, but also relations of interaction which are expressed in a 'qualitative' manner. A classic example of this last case is represented by the Machian analysis of the experiment of Newton bucket: in fact when Mach considers the case of the fixed stars in 'rotation' and the vessel stopped, assuming the presence of centrifugal forces in the water contained in the bucket, he is proposing - although not so clear - a functional relationship between the fixed stars and the water, and this feature is only 'qualitative' and not quantitative. It will be the task of Einstein and other followers of Mach's thought trying to give 'quantitative' form to this insight.

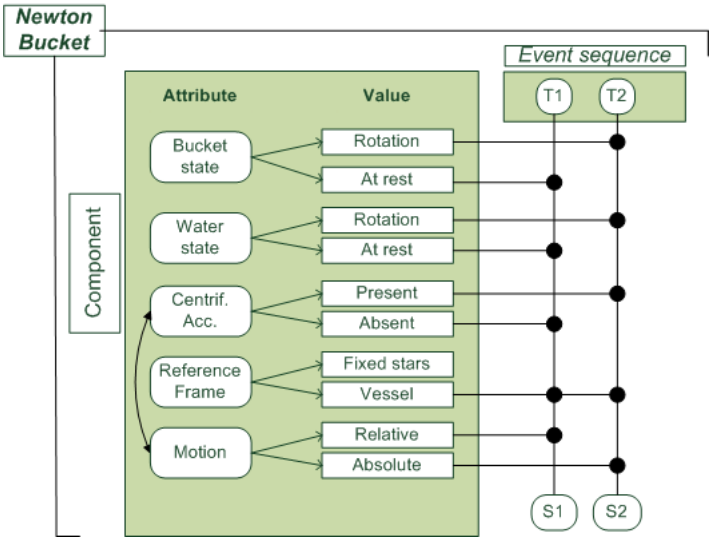
Finally it is to be observed regarding the dynamic frame just built, that the choice of the values of the attributes completely determines also the type of relationship that exists between them; so for example if you choose pressure, temperature and volume of a perfect gas as values of the 'relative' attributes, the '*relation type*' is determined as FUNCTIONAL.

Once it is clear what is meant by 'relative' in the Mach's work, it is useful to analyze the Newton bucket experiment using the '*event concept*' introduced in the previous paragraphs. To begin remember that the experiment of the bucket was used by Newton to discern between relative and absolute motion and below we report his observations present in the Scholium:

If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then lled with water, and held at rest together with the water; after, by the sudden action of another force, it is whirled about in the contrary way, and while the cord is untwisting itself, the vessel continues for some time this motion; the surface of the water will at rst be plain, as before the vessel began to move; but the vessel by gradually communicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little, and ascend to the sides of the vessel, forming itself into a concave gure [. . .] This ascent of the water shows its endeavour to recede from the axis of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, discovers itself, and may be measured by this endeavour. [. . .] And therefore, this endeavour does not depend upon any translation of the water in respect to ambient bodies, nor can true circular motion be dened by such translation

According to the British scientist the presence of centrifugal accelerations in the water when the bucket is in rotation - and expressed by the concave shape of the surface of water - were to indicate the presence of an absolute motion of the water. In fact, how does it explain the concave shape of the water when you consider the bucket in rotation? For an observer positioned on the bucket the water is at rest when the bucket is stopped, but also when the bucket is in motion.

The vessell experiment is represented by the following frame:



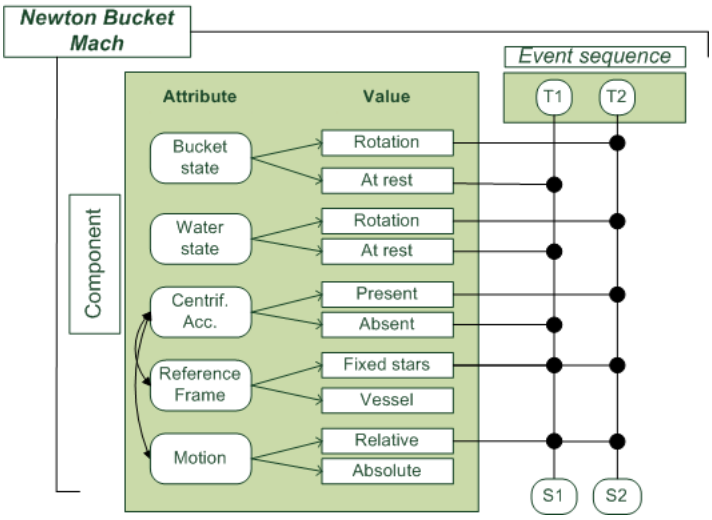
where we can highlight that in the component frame are present a series of attributes that represent the state of bucket, of water, the presence of centrifugal acceleration, the reference system and finally the type of motion.

As we expect the event sequence consists only of two instants identified by the state of motion of the bucket. It should also be noted that for Newton the presence of centrifugal acceleration uniquely determines the state of motion; from this point of view, we are in the presence of an *attribute constraints* (**valutare se è piuttosto un vincolo sui valori assunti**). Finally we observe that for Newton the 'physical' reference system which is used during the reasoning is the bucket, while the fixed stars are not considered at all (what is proposed by Mach as we shall soon see).

If the above is the event concept for the bucket experiment proposed by Newton, we can now analyze and propose what is the event concept based on Mach's considerations as is reported in the quotation (1) that we now write again:

> "*Newton's experiment with the rotating vessel of water simply inform us, that the relative rotation of water with respect to the sides of the vessel produce no noticeable centrifugal force, but that such forces are produced by its relative rotation with respect to the mass of the earth and other celestial bodies*"

The frame is the following:

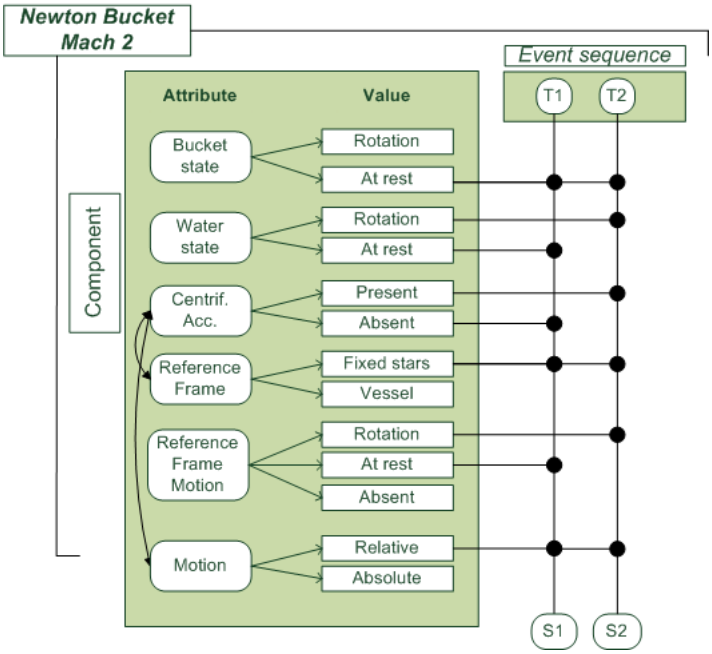


One aspect to be highlighted is that in the passage between the event concept of Newton and that of Mach there is a substantial 'structural stability' [37] which facilitates the comparison between the two. The difference lies in the assessment of the reference system: what for Newton was not significant for Mach now - in the light of his philosophical position - becomes imporant. The centrifugal accelerations are valued compared to the sphere of the fixed stars and not in relation to the sides of the bucket. Note also that the presence / absence of inertial forces determines not only the type of motion, but also the reference system.

In addition to the simple interpretation of the bucket experiment, Mach also made some assumptions that lead to think of a causal mechanism that determines the inertial properties of the water in the vessel: a classic example is the following passage:

> "*Try to fix the newtonian vessel and rotate the sphere of fixed stars and then prove the absence of centrifugal forces*"(2)

from which it is easy to deduce that the centrifugal acceleration in the water would occur even in the case in which the vessel remains stationary, while the stars make a revolution around it. We can build an event-concept also for this hypothetical experiment suggested by Mach: unlike the previous two is necessary to introduce an *attribute* relating to the type of motion of the reference system. With this simple trick you have the following dynamic frame:

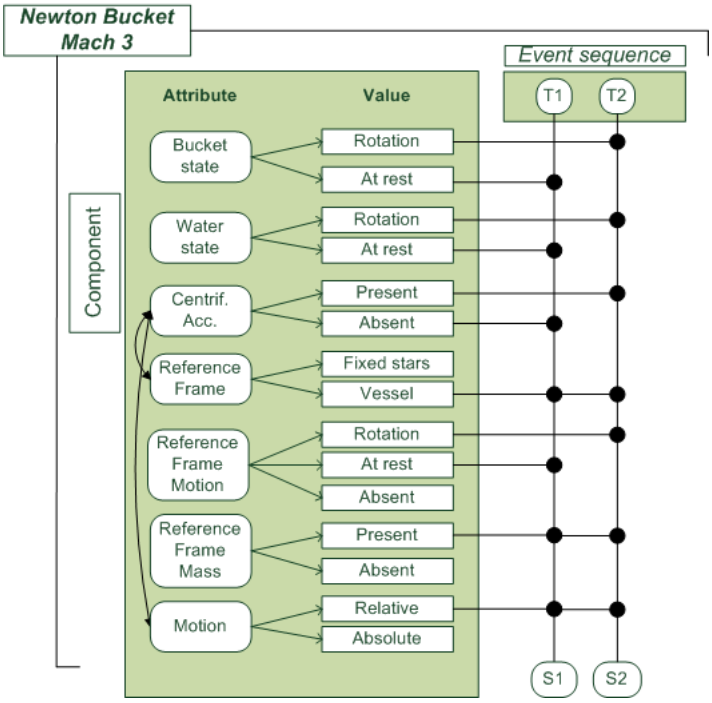


where it is observed that at time T_2 the centrifugal accelerations are present in the water, which they are valued in reference to the sphere of the fixed stars and that the latter are in rotation with respect to the bucket. We can make a similar reasoning also for another Machian observation which aroused much interest over the time and that suggests the existence of a causal mechanism as in the quotation (3):

> "*No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick*" (3)

In this case the mass and the dimensions of the walls of the bucket come into play - although the former has greater significance - and this last returns to have

the characteristics of the reference system. Although the expression of Mach is a warning to not exceed the limits of experience, it is easy to consider (3) as a mental experiment to be performed; in this respect it should be remembered that Einstein's article of 1912 [11] considers just a spherical shell of matter that revolves around a massive particle inside it. This third passage can be transferred in a dynamic frame by adding an extra '*attribute*' relative to the mass of the reference system, which leads to the following event-concept:



It is interesting to note that in the last Mach's observation intervenes for the first time a dynamic characteristic such as the mass that can determine the presence of the centrifugal accelerations in the water; in the previous event-concept this aspect was not present.

Although the four event-concept have a '*structural stability*' as we have already said, it may be interesting to determine the similarity projecting their attributes in a multidimensional space - as was done for the taxonomy of Sundeval. Let's just consider the concepts S_2 of the preceding event-concepts - as they are the most significant - and associate the individual attributes to a size of 7-dimensional space [11]; so for example if we consider the attribute 'buckets state', it can have the values AT REST and ROTATION that are mapped in the values $\{-1, 1\}$. In this way the subconcepts S_2 are represented by the vectors:

$$S_{2N} = (1, 1, 1, -1, 1, -1, -1)$$

[11] We have considered the seven '*attributes*' of the last event-concept

$$S_{2M1} = (1,\, 1,\, 1,\, 1,\, -1,\, -1,\, 1)$$

$$S_{2M2} = (-1,\, 1,\, 1,\, 1,\, 1,\, -1,\, 1)$$

$$S_{2M3} = (1,\, 1,\, 1,\, -1,\, 1,\, 1,\, 1)$$

that produces the distances:

$$d\,(S_{2N},\, S_{2M1}) = \frac{S_{2N} \cdot S_{2M1}}{\|S_{2N}\|\|S_{2M1}\|} = 0.020$$

$$d\,(S_{2N},\, S_{2M2}) = \frac{S_{2N} \cdot S_{2M2}}{\|S_{2N}\|\|S_{2M2}\|} = 0.020$$

$$d\,(S_{2N},\, S_{2M3}) = \frac{S_{2N} \cdot S_{2M3}}{\|S_{2N}\|\|S_{2M3}\|} = 0.061$$

$$d\,(S_{2M1},\, S_{2M2}) = \frac{S_{2M1} \cdot S_{2M2}}{\|S_{2M1}\|\|S_{2M2}\|} = 0.061$$

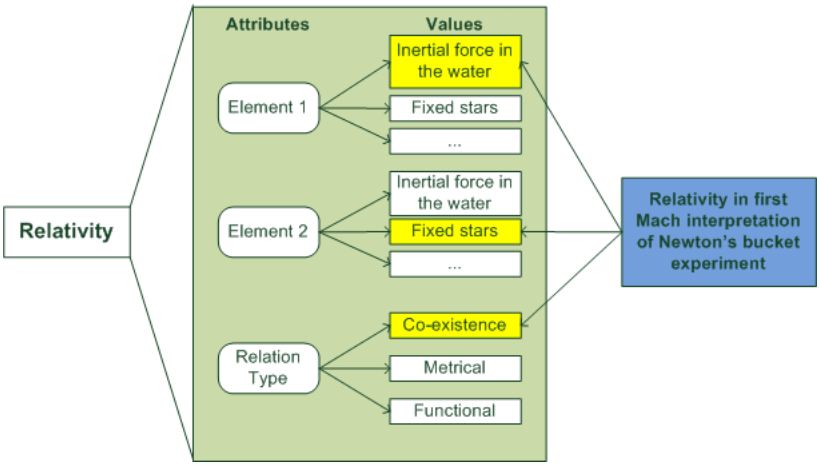
$$d\,(S_{2M1},\, S_{2M3}) = \frac{S_{2M1} \cdot S_{2M3}}{\|S_{2M1}\|\|S_{2M3}\|} = 0.020$$

$$d\,(S_{2M2},\, S_{2M23}) = \frac{S_{2M2} \cdot S_{2M3}}{\|S_{2M2}\|\|S_{2M3}\|} = 0.020$$

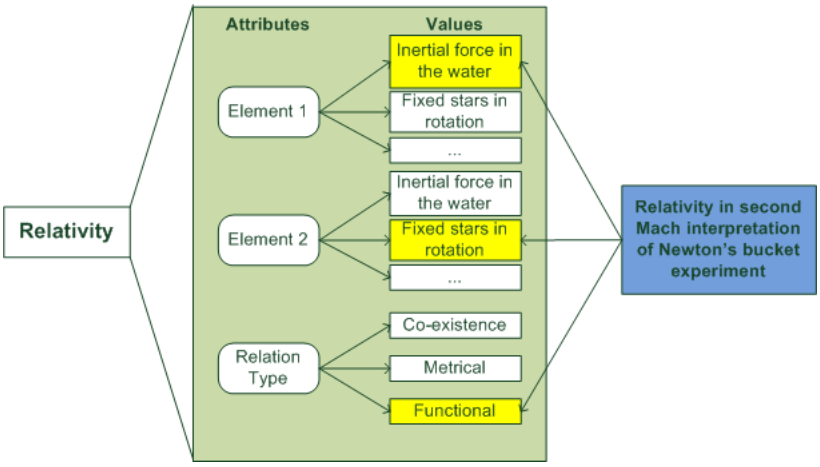
where instead of the Euclidean distance it is used the '*cosine similarity*' [12]which is generally used in Information Retrieval to compare two vectors. Note that more the calculated value is high and greater is the similarity between the two concepts. With this simple observation, it is clear that the similarities between the concept S_2 in the event-concept of Newton and the one corresponding to the first and second Mach's observation (quote (1) and (3)) is very low and there is no difference when you consider the first or second observation. Instead there is a greater similarity with the third observation (quote (3)), because the experiment is essentially the same; there is 'only' the additional consideration on the mass of the reference system. Finally, if we compare the three Mach's observations among one another, we obtain that the first two are very similar, while the difference between the first two and the third is definitely higher.

[12]The cosine similarity is defined as $\cos\theta = \frac{A \cdot B}{\|A\|\|B\|}$

To complete the analysis of the bucket experiment remains to determine the dynamic frame corresponding to the 'relative' concept and its values. If we consider the event-concept on the quote (1) we see that the elements that are involved are the inertial forces and the presence of the fixed stars. It's easy to observe that the 'relation type' is of type CO-EXISTENCE because Mach simply observes that the centrifugal accelerations in the water are in reference to the sphere of the fixed stars. These observations can be reproduced in the following dynamic frame:



Instead the dynamic frame corresponding to the quote (3) is very different, where the elements involved in the concept of relativity are always centrifugal acceleration in the water and the rotation of the fixed stars; in this case, however, the attribute 'relation type' takes the value FUNCTIONAL since it establishes a 'causal' relationship between the first and the second element. It is undoubtedly true that the value COEXISTENCE could also be used, but the value FUNCTIONAL includes in it also the simplest relation type.



We should also discuss the dynamic frame for the last Mach's sentence (the (3)), but in hindsight it corresponds to the previous one; rather it strengthens the functional feature as well as besides the movement of the reference system (in this case the bucket and not the fixed stars) it is introduced - as already stated - a further dynamic aspect that is the bucket mass.

4 Conclusions

In this article we used the dynamic frame to clarify the concept of 'relative' in Machian work. We pointed out that it can be characterized by three types of 'relation type' and that this aspect is determined from the group of elements that is taken into account. We then analyzed the experiment of Newton's bucket and the comments to it provided by Mach, creating each time the event-concept that highlighting the most significant aspects. Dipping then this information in a multidimensional space we have also tried to understand the similarities between the various concepts involved. Finally, we have specified the concept of relative to the event-concept corresponding to Mach's observetions, showing how in the most controversial sentences the relation type is functional.

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