Remarks on 1912 Einstein's note[1]

In 1912 Einstein published a series of articles on the static gravitational field that end with a note on an hypothetical gravitational effect like that of electromagnetic induction: in this note, for the first time, there are some remarks that resume the machian analysis on inertia. In the "Mechanics" Mach analysed the foundations of Classical Physics and in some pages seemd to suggest something very unusual: the inertial properties of a body are in some manner linked to the presence of the other masses of the universe. In 1912 article Einstein considers a spherical shell K with mass M, a massive point P inside it, and wonders if the acceleration of K could determine the inertial behaviour of P.

Einstein starts from the consideration that gravitational energy is the same as that of Special Relativity; this is a result which Einstein already reaches in 1907, trying to extend the actual theory beyond its limits (see the note "*Einstein results on 1907*"[1]). Infact in the first point of the article he underlines that "*According to the theory of relativity, the inertial mass of a closed physical system depends on its energy content in such a way that an increase of the energy of the system by E will increase the inertial mass by* E/c^2*...* " and then he notes that if M is the inertial mass of the shell without the massive point and m is the mass of P in absence of K, we have that the total mass of the system, whereas the two objects are very distant, is the sum $(M+m)$. But we are in the condition with P inside K, and so we have to evaluate the fact the two masses are close. Bearing in mind from the one hand that gravitational energy needed to carry P at infinity is equal to kMm/R – where k is the universal costant of gravitation and R is the radius of the spherical shell – and from the other that Einsten considers gravitational energy as any energy, we can conclude inertial mass of the system with P at the center of K has the form:

$$M + m - \frac{kMm}{Rc^2} \cdots \tag{1}$$

This is the first point of Einstein reasoning.

After introducing the equations that describe the motion of a point particle inside a static gravitational field

$$\frac{d}{dt}\left[\frac{\frac{\dot{x}}{c}}{\sqrt{1-\frac{q^2}{c^2}}}\right] = \frac{-\frac{dc}{dx}}{\sqrt{1-\frac{q^2}{c^2}}} + \frac{R_x}{m}\ , \ \cdots \tag{2}$$

he proposes the Kinetic energy of P

$$L = \frac{m}{2} q^2 \frac{c_0}{c} \tag{3}$$

where q is the particle speed, c_0 is the speed of light at infinity – thought as its velocity in our gravitational potential – and c the speed of light inside the field

[1]Einstein, Albert, Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?, Vierteljahrsschrift für gerichtliche Medizin und öffentliches Sanitätswesen 44,(1912)[2]

created by the interaction between the shell K and the point P. So you need to determine the changing of light speed in each point of the space in order to know the right kinetic energy of the particle. Here it's evident another result reached by Einstein in the previous years: infact starting from 1911 he knows perfectly that the measured value of light speed depends on the gravitational field where the measurement is done, so finally is a function of spatial coordinates.

In order to obtain the function $c(x, y, z)$ Einstein approximates equations (2) for a point in slowly movement and subject only to gravitational field, obteining:

$$\ddot{x} = -c\frac{dc}{dx} \ , \ \cdots \tag{4}$$

The next step is to replace the body acceleration with the change of gravitational potential Φ along x axis.

$$\frac{d\Phi}{dx} = c\frac{dc}{dx} \ , \ \cdots \tag{5}$$

The integration of (5) is simple and give the result:

$$\frac{c}{c_0} = 1 - \frac{\Phi_0 - \Phi}{c_0^2} \tag{6}$$

Remains to evaluate the difference $(\Phi_0 - \Phi)$, but it equals $\frac{kM}{R}$ inside the shell and so finally the kinetic energy of point particle P has the form:

$$L_p = \frac{m}{2}q^2(1 - \frac{kM}{Rc_0^2}) \tag{7}$$

from which we deduces the value of inertial mass:

$$m' = m + \frac{kmM}{Rc_0^2} \tag{8}$$

After (8) Einstein notes we are faced with a result of high interest: *"It shows that the presence of the inertial shellK increases the inertial mass of the material point P inside the shell. This suggest that the entire inertia of a mass point is an effect of the presence of all other masses, which is based on a kind of interaction with the latter* " (there is a footnote in which Mach is cited as a forerunner of this idea).

To conclude we note that (8) is based on three fundamental concepts:

1. The gravitational energy is treated as any energy in Special Relativity
2. To first approximation equations (2) are true
3. It is assumed that light speed is not constant, but changes in a gravitational field

A last remark. In the searching of the inertial force acted on point$P$ if the shell is accelerated, Einstein admits from the beginning that this result is correct; infact he considers the force F acting on K as a linear combination of the two accelerations Γ and γ of K and P respectively..

References

[1] http://www.vielbein.it/pdf/articoli/risultatieinstein_1907.pdf.

[2] Einstein Albert. Gibt es eine gravitationswirkung, die der elektrodynamischen induktionswirkung analog ist? *Vierteljahrsschrift für gerichtliche Medizin und öffentliches Sanitätswesen*, 44:37–44, 1912.